Sharp and Smart's 11 stage, order 7 Runge-Kutta scheme with an order 6 embedded scheme

See: Explicit Runge-Kutta Pairs with One More Derivative Evaluation than the Minimum, by P.W.Sharp and E.Smart, Siam Journal of Scientific Computing, Vol. 14, No. 2, pages. 338-348, March 1993.

The nodes of the scheme are:

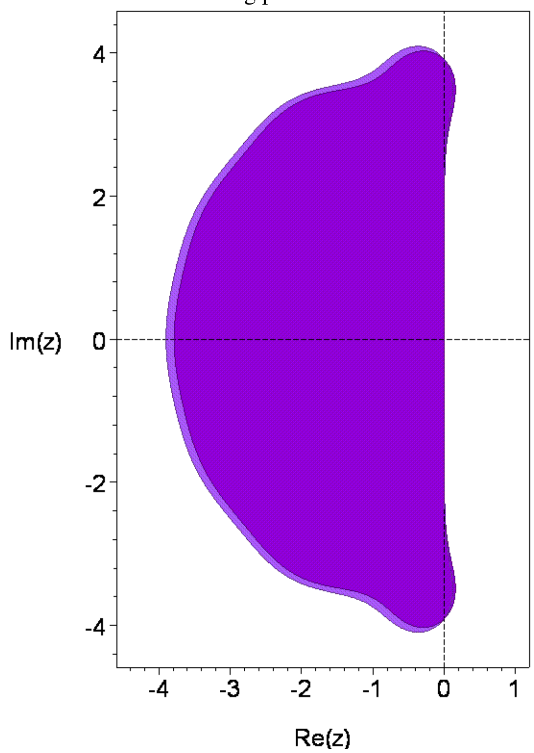
$$c_2=\frac{1}{50},\ c_3=\frac{27}{125},\ c_4=\frac{41}{100},\ c_5=\frac{57}{100},\ c_6=\frac{43}{50},\ c_7=\frac{2272510}{11977321},\ c_8=\frac{18}{25},\ c_9=\frac{5}{6},\ c_{10}=1,\ c_{11}=1.$$

The principal error norm of the order 7 scheme, that is, the 2-norm of the principal error terms is: $0.1274682565 \times 10^{(-4)}$.

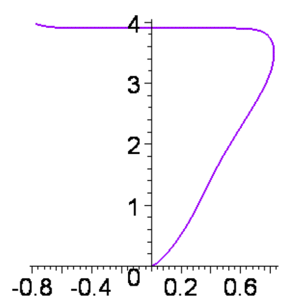
The 2-norm of the order 9 error terms is $0.3630580390 \times 10^{(-4)}$, which is approximately 2.848 times the principal error norm.

The principal error norm of the order 6 embedded scheme is: $0.1918150154 \times 10^{(-4)}$.
The maximum magnitude of the linking coefficients is: 10.06996058.
The 2-norm of the linking coefficients is: 20.83467890.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.
The real stability intervals of the order 7 and 6 schemes are respectively $[-3.89945, 0]$ and $[-3.7861, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 3.9069]$.

The coefficients in exact form are:
c[2]=1/50,
c[3]=27/125,
c[4]=41/100,
c[5]=57/100,
c[6]=43/50,
c[7]=2272510/11977321,
c[8]=18/25,
c[9]=5/6,
c[10]=1,
c[11]=1,

a[2,1]=1/50,
a[3,1]=-594/625,
a[3,2]=729/625,
a[4,1]=451/21600,
a[4,2]=0,
a[4,3]=1681/4320,
a[5,1]=19/160,
a[5,2]=0,
a[5,3]=361/3104,
a[5,4]=3249/9700,
a[6,1]=-31/200,
a[6,2]=0,
a[6,3]=520921/412056,
a[6,4]=-17371/11640,
a[6,5]=132023/106200,
a[7,1]=2595976687776897697659895773 6980/48759451412962829594551315 7189933,
a[7,2]=0,
a[7,3]=347890318302644246405985993187156250/132181740206709287575081822038851 9949,
a[7,4]=-171704697261714770949111645017 8750/746789492693272811158654361801 4237,
a[7,5]=297803047327251035777647517462 16250/258912687002832625147067486467854423,
a[7,6]=-302662548054389051180423185000/256628691647172787339743766942 07,
a[8,1]=42409705291266846/41646225640 7406875,
a[8,2]=0,
a[8,3]=3247095172038/883201854817,
a[8,4]=-518509279926/374238074075,
a[8,5]=43566922562973256 6638/393965828849029186615,
a[8,6]=-646869455911476 0/6194593900608963 7,
a[8,7]=-859375088109520617049100719450 2/321350454354555815090388058562 5,
a[9,1]=-1401024812030113404025/19887564677841032175639,
a[9,2]=0,
a[9,3]=13281373111234375/5150833217292744,
a[9,4]=-50491693720625/29100752640072,
a[9,5]=89097764687831645839731931 25/6271093223575470807674793192,
a[9,6]=-47923249417356350087 50/159776107397443897190271,
a[9,7]=-1532806290465891141166096531902118541769245/1203242011387872547807852011647420329982736,
a[9,8]=-7500029126894375/132689679447323376,
a[10,1]=3639303261543445061 2/3243905860948896634 25,
a[10,2]=0,
a[10,3]=-1462401427649331250/1547872145822482 11,
a[10,4]=4135780451822750/874504037187843,
a[10,5]=-23493787336470028952340089 50/1090914599757106529355865311,
a[10,6]=-7868660590842244375 0/52446632451499515953,
a[10,7]=2315079813491204524435067899365885119542372444358703/
3161690420395271575952352315737883080312607605842 00,
a[10,8]=-33473047374792524975/32907430028856870472,
a[10,9]=5594658687556280397846/18931898705209979401 75,

a[11,1]=25086077067018423630083/197875357745688550590720,
a[11,2]=0,
a[11,3]=-5122833329940625/508724268374592,
a[11,4]=13293920580875/2874148408896,
a[11,5]=-5991884647804937071374401618755/27727006417322986978460073273 6,
a[11,6]=-36014650553489237628498 75/2146128454918752594358208,
a[11,7]=606030238246181777051198920509497430523044409408159/
    747520501416409989678136744605131973486532880245 76,
a[11,8]=-1922750201834125/1941504226023936,
a[11,9]=12539348439579/3975412795840,
a[11,10]=0,

b[1]=771570009067/14036203465200,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=28304779228000000/53707434325074117,
b[6]=-296881060859375/515060733835389,
b[7]=744858303758379680905615939985761920312207508379/248722388447776459076443339652492214567388761840 0,
b[8]=-5118512171875/11763620626464,
b[9]=136801854099/127885521925,
b[10]=103626500437/1717635089268,
b[11]=0,

b*[1]=448234490819/8120946290580,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=7786773134600000/14452831163890377,
b*[6]=-408698637296875/567617951573694,
b*[7]=44267051503691526383253810782780678033 59/14828075230102658203818343670586143438076,
b*[8]=-5004542378125/10330679593521,
b*[9]=154806770859/124231649870,
b*[10]=0,
b*[11]=16/243.

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