

Tanaka-Yamashita Runge-Kutta scheme with an order 6 embedded scheme .. $c_8 \approx \frac{2006}{2007}$

See: On the Optimization of Some Nine-Stage Seventh-order Runge-Kutta Method, by M. Tanaka, S. Muramatsu and S. Yamashita, Information Processing Society of Japan, Vol. 33, No. 12 (1992) pages 1512-1526.

The nodes of the scheme are:

$$c_2 = \frac{36259}{463869}, c_3 = \frac{36259}{309246}, c_4 = \frac{36259}{206164}, c_5 = \frac{76401}{153188}, c_6 = \frac{5164663400901569152}{6688924124988083687}, c_7 = \frac{3486}{3517}, c_8 = \frac{44151}{44173}, c_9 = 1, c_{10} = 1.$$

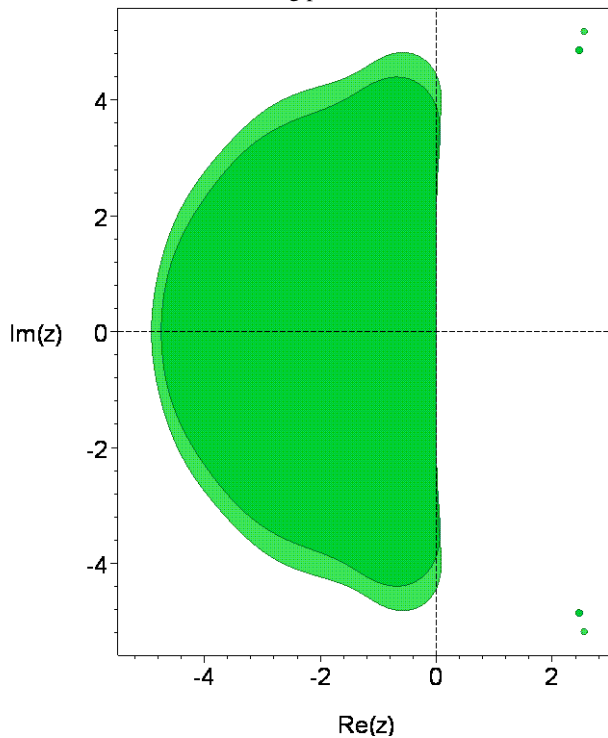
The principal error norm, that is, the 2-norm of the principal error terms is: $0.3243425994 \times 10^{(-5)}$.

The principal error norm of the order 6 embedded scheme is: $0.2182112630 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 457.6935657.

The 2-norm of the linking coefficients is: 1167.946960.

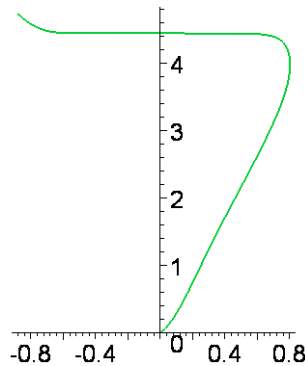
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6607, 0]$ and $[-4.7448, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 4.4361]$.

The coefficients in exact form are:

$c[2]=36259/463869,$
 $c[3]=36259/309246,$
 $c[4]=36259/206164,$
 $c[5]=76401/153188,$
 $c[6]=5164663400901569152/6688924124988083687,$
 $c[7]=3486/3517,$
 $c[8]=44151/44173,$
 $c[9]=1,$
 $c[10]=1,$

$a[2,1]=36259/463869,$
 $a[3,1]=36259/1236984,$
 $a[3,2]=36259/412328,$
 $a[4,1]=36259/824656,$
 $a[4,2]=0,$
 $a[4,3]=108777/824656,$
 $a[5,1]=17751533712975975593187/24112920357813127230992,$
 $a[5,2]=0,$
 $a[5,3]=-68331192803887602162951/24112920357813127230992,$
 $a[5,4]=7825717455900471140481/3014115044726640903874,$
 $a[6,1]=-2425518501234340256175806929031336393991001205323654593685210322030691047097621496102266496/$
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 $a[6,3]=126875939114499086848675646731069753055308007638565564293214808307459627250976287910912/$
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a[9,4]=6298831527954572673520838478029639446424615570453903300371170696118960335541193275024146681623960/
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a[9,6]=49192051734527182139396013466558216354763286834791148749699566514605553857954527798357018999449248197720
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a[10,1]=24861126512935523838485032295435745281790804119672244200744512677831357181363/
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a[10,3]=-76626859319946149305867456329803/167454524692981091214376557800,
a[10,4]=257532657386915224604779230484778835596042580268896440943054087972106955277512448850995064336363/
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a[10,5]=-103092665221253777021612043042409780416654274677686197534469014507504059634284484983141143/
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a[10,6]=1318945254307068672853031172410281620677291556423152759282406612372948205789241763483098989903852936890
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a[10,7]=-2175691361381933486174620849991740173349017185199505364607841/
482872625303278742130341621563226511344221688759361797916327450,
a[10,8]=-11327601987184122343710458559595782081610122892585097/
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a[10,9]=0,

b[1]=677260699094873524061210073954310211/13212228177645157882237395248920447488,
b[2]=0,
b[3]=0,
b[4]=5627843976805934592544586970647029617399366281651959837492864/
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b[7]=14322850798205614664394883796805489119964080948503151/
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b[8]=-16735096417960349589058935251250023138290806176584545269411/
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b[9]=33050288141543277444692395096256051/271248590133163812341791503489000,
b[10]=0,

b*[1]=962650826879437817605721930727384851/18874611682350225546053421784172067840,
b*[2]=0,
b*[3]=0,
b*[4]=99703652969826806275610089806158069716600653757297413344/
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b*[7]=179578338747395946570172802104016572846366090083599/
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b*[8]=-500374162579884236288722085953024481890963958534161489781/
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b*[9]=0,
b*[10]=80.

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