An order 7 Runge-Kutta scheme with an order 6 embedded scheme .. $c_8 = \frac{11}{12}$

The order 7 scheme considered here is similar to one constructed by Tanaka and Yamashita.
See: On the Optimization of Some Nine-Stage Seventh-order Runge-Kutta Method, by M. Tanaka, S. Muramatsu and S. Yamashita, Information Processing Society of Japan, Vol. 33, No. 12 (1992) pages 1512-1526.

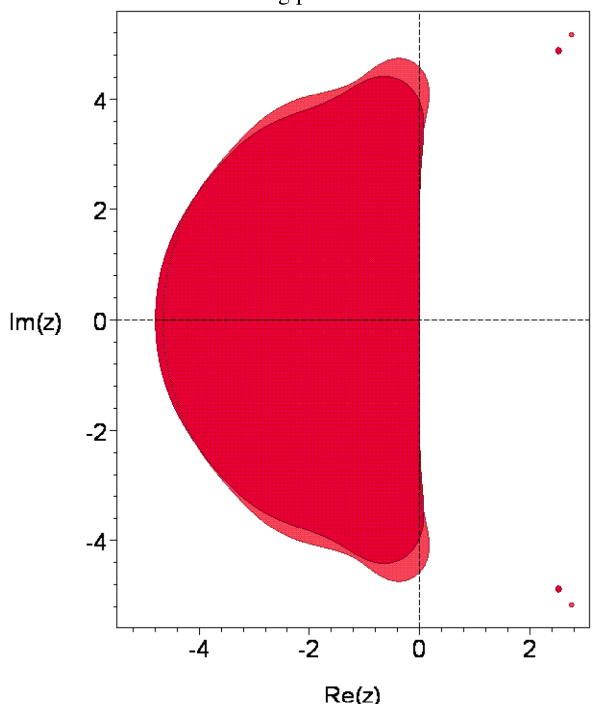
The nodes of the scheme are:

$$c_2 = \frac{8}{111},\ c_3 = \frac{4}{37},\ c_4 = \frac{6}{37},\ c_5 = \frac{307}{675},\ c_6 = \frac{31476021}{50816069},\ c_7 = \frac{277}{308},\ c_8 = \frac{11}{12},\ c_9 = 1,\ c_{10} = 1.$$

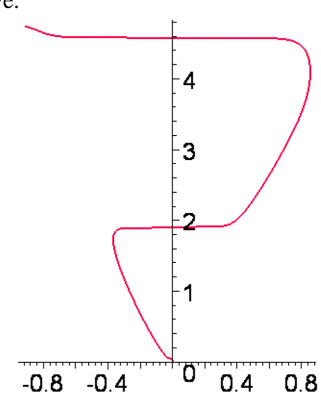
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1727361568 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.1609265372 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 50.87951814.
The 2-norm of the linking coefficients is: 105.0908421.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.
The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6607, 0]$ and $[-4.7936, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[1.9056, 4.5799]$.

The coefficients in exact form are:
c[2]=8/111,
c[3]=4/37,
c[4]=6/37,
c[5]=307/675,
c[6]=31476021/50816069,
c[7]=277/308,
c[8]=11/12,
c[9]=1,
c[10]=1,

a[2,1]=8/111,
a[3,1]=1/37,
a[3,2]=3/37,
a[4,1]=3/74,
a[4,2]=0,
a[4,3]=9/74,
a[5,1]=7187177921/11071687500,
a[5,2]=0,
a[5,3]=-4606608373/1845281250,
a[5,4]=25488039817/11071687500,
a[6,1]=-25155061428355951801474560458369199/8188467530271553334690238468952588,
a[6,2]=0,
a[6,3]=165887544885608390859549696826251/13336266335947155268225144086242,
a[6,4]=-1882228421763731849935929306392999001/194949541298875515710915156252685556,
a[6,5]=13579013110155450058895015284128000000/14962377294688695830812738242393616423,
a[7,1]=197761222523152422894366588349/22337214752609007269401861632,
a[7,2]=0,
a[7,3]=-2634239197724068023/73383811330407424,
a[7,4]=853339267965919636844757871233901/29050523840824274640853346171904,
a[7,5]=-8076034988531398673856692417953125/3253783652546328667438519984062464,
a[7,6]=123649325987132387040525516232779481466261215/117719554361990101551344966893015826124767232,
a[8,1]=107753957259980585980666846829/9233074231278477689097027072,
a[8,2]=0,
a[8,3]=-568074707723/11955981312,
a[8,4]=42472998581230749927225480551001035/1092556099588534076090378381976576,
a[8,5]=-517989056407037173427474451800390625/149577483273160744135700462783430656,
a[8,6]=634696345409674948643814960063972951409491091120921/
461800006308941492267876194538077346249461944287232,
a[8,7]=-76885773888994425792394496/2936131073221001350750428267,
a[9,1]=272438987179417236599886503/22132409363268856714696212,
a[9,2]=0,
a[9,3]=-23372286858163/459365334286,
a[9,4]=198243212868288076356622474962659/4716914193773010520669300917060,
a[9,5]=-1363936981682510968476522363890625/327473641455662189001538729784746,
a[9,6]=25755888893946773752332721682348547077412933629363765/
16329311478602547059307390176423181582188873526860658,
a[9,7]=1319269904346934743230785049600/1396320388726418818204188765431,
a[9,8]=-14178287818409238951533568/17392109352684914430075245,
a[10,1]=4235442918959811567229357361/2644891055834980588022585880,
a[10,2]=0,
a[10,3]=-786113211/114163016,
a[10,4]=13413911790729567706817626177847323/2184282349709064708329559644956425,
a[10,5]=-2262780050068551846381323571012571875/4007332691519425639276178218750621696,
a[10,6]=2105989008224777915652413334829351288879837515690767182151/
4995591662532871431019909905272284678024887162604993075200,
a[10,7]=961294347572182289791507536/906213294204012762577292675,
a[10,8]=-3224676619809072/4164728781219325,

a[10,9]=0,

b[1]=9006801331565/192721633186968,
b[2]=0,
b[3]=0,
b[4]=50540265241285806109627/197357275435749536626200,
b[5]=144779348708219481591796875/537274095568517141694152704,
b[6]=413443953067924177634578015801217648888914981481891/
2707658017012075759838783851383081879913377991065600,
b[7]=152166844605803938484112/146056286076938331608425,
b[8]=-323384822090352/378611707383575,
b[9]=229682667143/2630603575040,
b[10]=0,

b*[1]=1447026424436341/31799069475849720,
b*[2]=0,
b*[3]=0,
b*[4]=4978446866776507926899/19099091171201568060600,
b*[5]=1463416045449156087890625/5839935821396925453197312,
b*[6]=4019886664009688828764460732095930359371210969/
21840413770114935523161590954467164366214963200,
b*[7]=30522776918168741012576/42403437893304676918575,
b*[8]=-2149784413206048/4164728781219325,
b*[9]=0,
b*[10]=1/18.

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