An 11 stage order 7 Runge-Kutta scheme with a 12 stage FSAL order 6 embedded scheme .. $c_{10} = \frac{81}{82}$

The nodes of the scheme are:

$$c_2 = \frac{1}{100},\ c_3 = \frac{2}{15},\ c_4 = \frac{1}{5},\ c_5 = \frac{5120}{9991},\ c_6 = \frac{60}{97},\ c_7 = \frac{52}{313},\ c_8 = \frac{63}{157},\ c_9 = \frac{19}{23},\ c_{10} = \frac{81}{82},\ c_{11} = 1,\ c_{12} = 1.$$

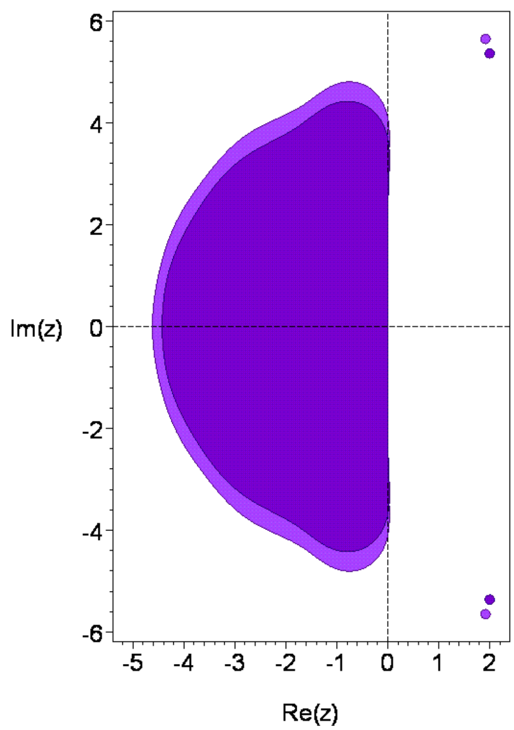
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1246313430 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.8223341109 \times 10^{(-4)}$.

The maximum magnitude of the linking coefficients is: 18.26986160.

The 2-norm of the linking coefficients is: 38.49824072.

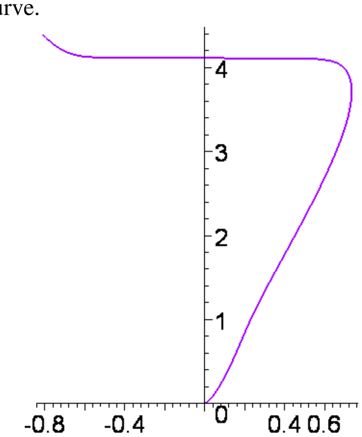
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6188, 0]$ and $[-4.4277, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 4.1087]$.

The coefficients in exact form are:
c[2]=1/100,
c[3]=2/15,
c[4]=1/5,
c[5]=5120/9991,
c[6]=60/97,
c[7]=52/313,
c[8]=63/157,
c[9]=19/23,
c[10]=81/82,
c[11]=1,
c[12]=1,

a[2,1]=1/100,
a[3,1]=-34/45,
a[3,2]=8/9,
a[4,1]=1/20,
a[4,2]=0,
a[4,3]=3/20,
a[5,1]=551874974720/997302429271,
a[5,2]=0,
a[5,3]=-2086699008000/997302429271,
a[5,4]=2045902848000/997302429271,
a[6,1]=70365/1204352,
a[6,2]=0,
a[6,3]=0,
a[6,4]=15000000/48955027,
a[6,5]=16390905/64600448,
a[7,1]=21278500033609/276420238876800,
a[7,2]=0,
a[7,3]=0,
a[7,4]=9893050830728000/91236732526195641,
a[7,5]=-2901059421298252903/76225368652427491200,
a[7,6]=436379361176824/23234416797464775,
a[8,1]=627197176791399348709/22893533288085131673600,
a[8,2]=0,
a[8,3]=0,
a[8,4]=-26470464436388981652125/236136288092530359764076,
a[8,5]=825734282216068905765330677/6313097845989067825180262400,
a[8,6]=-49138310819605862695843/962154393150421608110400,
a[8,7]=13/32,
a[9,1]=-2805292064095175105994189803671936576349564929/
195439700408341869784184509846462968849917952000,
a[9,2]=0,
a[9,3]=0,
a[9,4]=-44489197025459151705444370840723674742716548825/
14932376340320772420636844398178733063223595932,
a[9,5]=-538125605455123224555096530708947439830683725706539/
221787063147752862838752824077163711222570059776000,
a[9,6]=5903957290670756919471851965687658329936372357281397/
4701038098831357101461987812303657884827097474792000,
a[9,7]=80638533780709324566519854325500435393/31664769935063948010841554250731537120,
a[9,8]=38023609700214727511244770779309498/15560366705131162570848900057690585,
a[10,1]=18745022701169741445732083919370592367818346398255185593/
43542944158840438871835940146729085397585013246181376000,
a[10,2]=0,
a[10,3]=0,

a[10,4]=174955774658845813020015026443375837215431521574722425/
1298801282439065129217546001415326003523811103095855 2,
a[10,5]=72703899173100547975701530898752038618015765157471787669321/
6944684059165807161861542521065278114605347407607709696000,
a[10,6]=-18513454683442942614776350098381846385951171415826406124378 67/
420703407864684992323487936970444634267986659475521049568000,
a[10,7]=-142383111687918201861880767560889949537519404224643/
13608444565909841005179111919047260723369140257280,
a[10,8]=-15762011599975752003382750329296637334838001/170538268612761013476551976905055472269248 0,
a[10,9]=1570980729807122133129202485/21707495839446896008350129 92,
a[11,1]=41340133965442600475649228072204707500441610405588 3/
719868089323382472727694115370573342900039059456000,
a[11,2]=0,
a[11,3]=0,
a[11,4]=4068247489923915770015231221964449550938389658325/
222675331628898775507843272599152181481922934868,
a[11,5]=42270128817945130949585752844996768168386895932329559 1/
2976609753242264616837849775985514087041113675161600 0,
a[11,6]=-647563362956678228844742602772133226055290595664033985999 43/
105920343337637039706449916975617887359269147304227520720 00,
a[11,7]=-2933204189652449798399861612546549582225310986953/
205013581499999542677439978824334676268210882720,
a[11,8]=-62479747560850016049359399624970903878422 6/497736028930361526506167928459999015164 65,
a[11,9]=386553003394743260 88/408008631369867792 55,
a[11,10]=-683002535143417196569 6/369666731905701279737985,
a[12,1]=6470068411/127052452800,
a[12,2]=0,
a[12,3]=0,
a[12,4]=0,
a[12,5]=0,
a[12,6]=14036525372223254273/576294203941527456 00,
a[12,7]=73543228807230391791 31/302164288651610906976 00,
a[12,8]=22370757398626528948 3/110157542907766000920 0,
a[12,9]=5315253908682019/32851416704215200,
a[12,10]=347047251949199216/1213217205998680575,
a[12,11]=-95980357/508344480,

b[1]=6470068411/127052452800,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,
b[6]=14036525372223254273/57629420394152745600,
b[7]=735432288072303917913 1/302164288651610906976 00,
b[8]=22370757398626528948 3/110157542907766000920 0,
b[9]=5315253908682019/32851416704215200,
b[10]=347047251949199216/1213217205998680575,
b[11]=-95980357/508344480,

b*[1]=4575937828684274263448 21/883669918237518799296 0000,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=0,
b*[6]=911082200138648538937372713121003/400821740041027803911633227392 0000,
b*[7]=503904010603015936847352266508232049/2101600452117104186799675302680320000,
b*[8]=16283245430995038704640712154556497/766163145923551590977348731454400 00,
b*[9]=418537996431230550055107156803/22848680268060519559769846400 00,

b*[10]=2137829181363408719740436246579/10547647109544872417769030948750,
b*[11]=-4366466929950751198793/35356163157685532736000,
b*[12]=1/160.

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