An 11 stage order 7 Runge-Kutta scheme with a 12 stage FSAL order 6 embedded scheme .. $c_{10} = \frac{32}{33}$

The nodes of the scheme are:

$$c_2 = \frac{1}{100},\ c_3 = \frac{4}{27},\ c_4 = \frac{2}{9},\ c_5 = \frac{11417}{21528},\ c_6 = \frac{49}{78},\ c_7 = \frac{47}{283},\ c_8 = \frac{30}{73},\ c_9 = \frac{37}{46},\ c_{10} = \frac{32}{33},\ c_{11} = 1,\ c_{12} = 1.$$

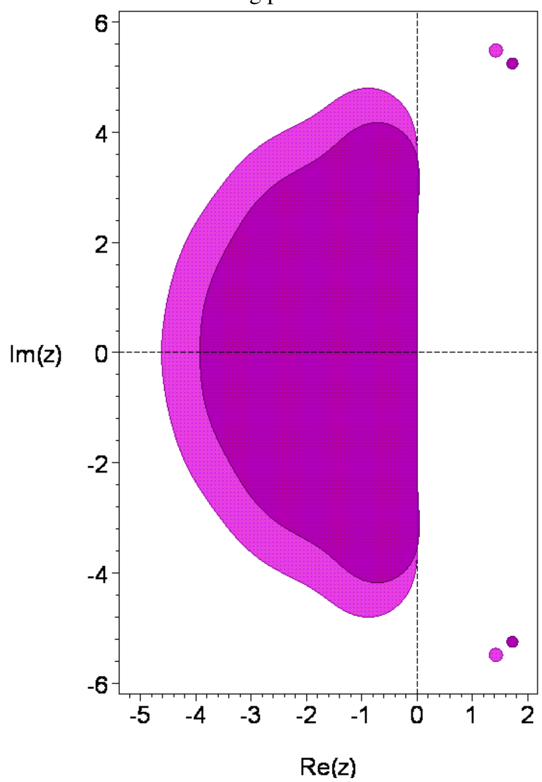
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1426525824 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.1084068672 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 10.56065553.

The 2-norm of the linking coefficients is: 18.99484408.

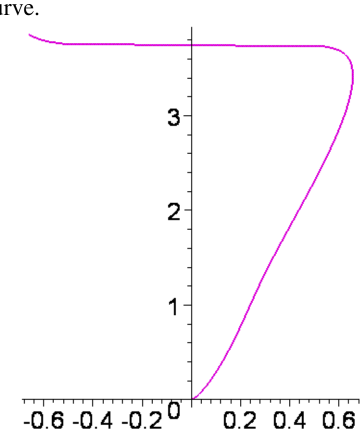
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6176, 0]$ and $[-3.9227, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 3.7489]$.

The coefficients in exact form are:
c[2]=1/100,
c[3]=4/27,
c[4]=2/9,
c[5]=11417/21528,
c[6]=49/78,
c[7]=47/283,
c[8]=30/73,
c[9]=37/46,
c[10]=32/33,
c[11]=1,
c[12]=1,

a[2,1]=1/100,
a[3,1]=-692/729,
a[3,2]=800/729,
a[4,1]=1/18,
a[4,2]=0,
a[4,3]=1/6,
a[5,1]=451814999377/985407860736,
a[5,2]=0,
a[5,3]=-552805397249/328469286912,
a[5,4]=96066394193/54744881152,
a[6,1]=131173/1890096,
a[6,2]=0,
a[6,3]=0,
a[6,4]=5764801/17935632,
a[6,5]=4769464/20091357,
a[7,1]=1176539682457387/14353367828755172,
a[7,2]=0,
a[7,3]=0,
a[7,4]=17717342174781/163306752068660,
a[7,5]=-528062841353883264/10338013178660912633,
a[7,6]=1679528554701504/62911488820241285,
a[8,1]=110903909289926737793/4320958290769994040132,
a[8,2]=0,
a[8,3]=0,
a[8,4]=-11607238985710021183/180261023052662924020,
a[8,5]=8332987977461879955626624/57056453830329950469093005,
a[8,6]=-397914630242458427722/6312988060727977701695,
a[8,7]=11/30,
a[9,1]=-199116158962708264981792850380273725319682574083/212209570379395954037673565209665038844524173920,
a[9,2]=0,
a[9,3]=0,
a[9,4]=-902399027478634049318468354365831042971264299/376720159407433656819366352813041281670901920,
a[9,5]=-1932841049216489668013274675445061103749096456/1225034474225538046212066937516775761976271555,
a[9,6]=61212379964740003623393766579069964177763490780547/67292203490497755527349011795764425282435991108860,
a[9,7]=1767352010455801522759919865162797424677/853631298904311486745705433404624991760,
a[9,8]=48803138404357581919875739936453/25803619795436169827228351014320,
a[10,1]=146812882912103498223477710702327669160817645499822332/
347136327550264350281019050691094921672653636486171345,
a[10,2]=0,
a[10,3]=0,
a[10,4]=8380907279399940341415520095001429689250499016988/1665529140598002776815163790462148830898906131435,
a[10,5]=6459917812385485497254681791738665918061301836694665216/
2108701812611736155789874398208067694599974390240225435,
a[10,6]=-329623101936975913713940238539594299201957668540750512/
216369193273123311533898096852817531450154842258383315,

a[10,7]=-239812294208127777232571199557076199094738401624/62716593235272943126739970926235455172660665115,
a[10,8]=-19209909490782204629345787487654324166296/6932257046340021610507379175381008191035,
a[10,9]=210642399729505665392/369807601102457012649,
a[11,1]=221939233494467949726864332982580511239150575217/255387155540276753091910917079234490443699161270,
a[11,2]=0,
a[11,3]=0,
a[11,4]=181163263192487297546324132938257205801916729/17154547141864477699072019121010975163450940,
a[11,5]=5035138254118716342864368937874683246617259598976/775682801056824859328708920179990438628643868105,
a[11,6]=-77224801754835239618997430260090636543172193252536784/
20265242890170705812536717982284413527106230272155465,
a[11,7]=-2041746049322766275834287576167164751357072803/242398642004974788232613894826395428361025315,
a[11,8]=-69880579855847721894868113496992143638/12013145933881997013005874021379315455,
a[11,9]=667711875409843102380/547572557872664725429,
a[11,10]=-1808859196027797903/20206200178110899692,
a[12,1]=2354923/46428480,
a[12,2]=0,
a[12,3]=0,
a[12,4]=0,
a[12,5]=0,
a[12,6]=31816481086962/143829479014165,
a[12,7]=7273629826007016071/29687570322557889840,
a[12,8]=131953911267751793/602866646471570040,
a[12,9]=29769129062566/205676274838545,
a[12,10]=30049446213369/166412182583360,
a[12,11]=-67996657/1112423760,

b[1]=2354923/46428480,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,
b[6]=31816481086962/143829479014165,
b[7]=7273629826007016071/29687570322557889840,
b[8]=131953911267751793/602866646471570040,
b[9]=29769129062566/205676274838545,
b[10]=30049446213369/166412182583360,
b[11]=-67996657/1112423760,

b*[1]=182127300329983916501/3507184399433109696000,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=0,
b*[6]=285550402807929509723935449/1482899423367801818133032950,
b*[7]=6443295529403196642395281317251/26849338691229488574870087988000,
b*[8]=143483908435216987345410922193/615408757915355446585175892000,
b*[9]=139711798081481043314568703/776834200093629660855997950,
b*[10]=2259083734163473774459617/17881499156003594882624000,
b*[11]=-2987435988158249671/113556675995246584800,
b*[12]=1/400.

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