

An 11 stage order 7 Runge-Kutta scheme with a 12 stage FSAL order 6 embedded scheme .. $c_{10} = \frac{15}{16}$

The nodes of the scheme are:

$$c_2 = \frac{1}{16}, c_3 = \frac{4}{25}, c_4 = \frac{6}{25}, c_5 = \frac{915}{1696}, c_6 = \frac{5}{8}, c_7 = \frac{35}{209}, c_8 = \frac{29}{72}, c_9 = \frac{11}{13}, c_{10} = \frac{15}{16}, c_{11} = 1, c_{12} = 1.$$

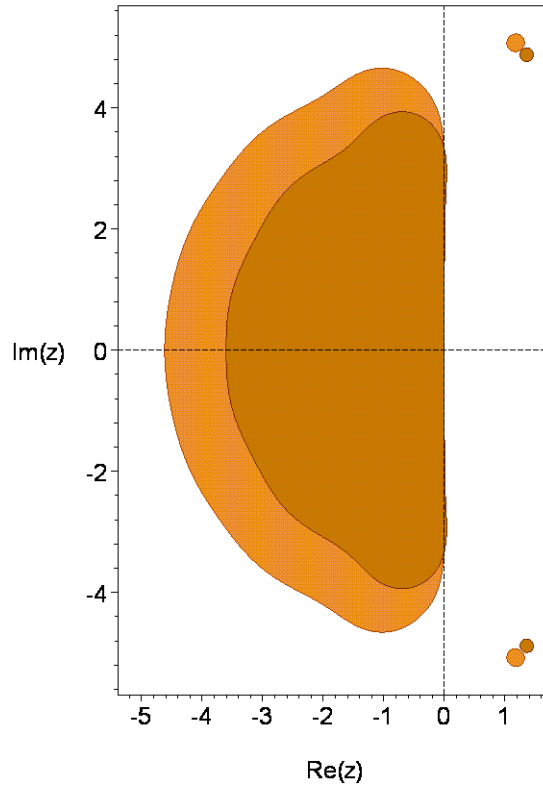
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1697875240 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.1327775571 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 10.36978177.

The 2-norm of the linking coefficients is: 19.19076853.

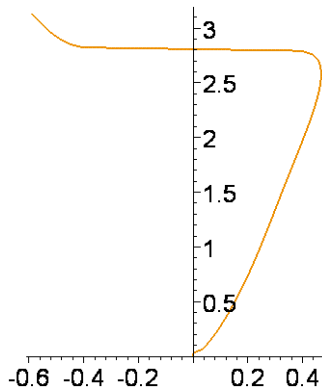
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6199, 0]$ and $[-3.6054, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 2.8067]$.

The coefficients in exact form are:

$c[2]=1/16,$
 $c[3]=4/25,$
 $c[4]=6/25,$
 $c[5]=915/1696,$
 $c[6]=5/8,$
 $c[7]=35/209,$
 $c[8]=29/72,$
 $c[9]=11/13,$
 $c[10]=15/16,$
 $c[11]=1,$
 $c[12]=1,$

$a[2,1]=1/16,$
 $a[3,1]=-28/625,$
 $a[3,2]=128/625,$
 $a[4,1]=3/50,$
 $a[4,2]=0,$
 $a[4,3]=9/50,$
 $a[5,1]=15090136995/39027212288,$
 $a[5,2]=0,$
 $a[5,3]=-53100995625/39027212288,$
 $a[5,4]=29533111875/19513606144,$
 $a[6,1]=32755/421632,$
 $a[6,2]=0,$
 $a[6,3]=0,$
 $a[6,4]=9765625/29258496,$
 $a[6,5]=1488770/6971751,$
 $a[7,1]=1072887507055/12570100065468,$
 $a[7,2]=0,$
 $a[7,3]=0,$
 $a[7,4]=1075426748046875/9595107694235844,$
 $a[7,5]=-26411117095731200/385766943434163819,$
 $a[7,6]=23430395520/608661493759,$
 $a[8,1]=232418455423819/11076629098233600,$
 $a[8,2]=0,$
 $a[8,3]=0,$
 $a[8,4]=-232407582514046875/2367427911414660864,$
 $a[8,5]=881809586743097626/5311459993015086075,$
 $a[8,6]=-14411638756714/175988393502075,$
 $a[8,7]=19/48,$
 $a[9,1]=-8046942294592811005033672093485229/31693413191384942205414737020389900,$
 $a[9,2]=0,$
 $a[9,3]=0,$
 $a[9,4]=-536231038652840856428143497005609375/138242660274515884207093807951326924,$
 $a[9,5]=-312405090743882815734153055386047660032/138949582969101476796789174014714391225,$
 $a[9,6]=46374757254514641197506707162751264/33542814156299871502725237873062925,$
 $a[9,7]=2051784331177468089852258884/674022310351068617436737421,$
 $a[9,8]=1762562147235459382311840/629339225351137831406851,$
 $a[10,1]=215720375695479264063749229806841635/440720278308021223336256911460990976,$
 $a[10,2]=0,$
 $a[10,3]=0,$
 $a[10,4]=12320199720948281040487332306693759765625/2354898495606471238613818261942735405056,$
 $a[10,5]=1866488369998347663233388794677364275/6604186914582246254039171103542098296,$
 $a[10,6]=-78458209082510636098652961448851403235/65709391590090207335818975096879372288,$
 $a[10,7]=-284269998655149797778753043937075/78560914257412947734286452523008,$
 $a[10,8]=-40415483223195085710169801875/13000342692048285899311310848,$
 $a[10,9]=1620538162595625/5204697537445888,$

a[11,1]=-975767605592872353548579494188313/1054764737809245464809448652511500,
a[11,2]=0,
a[11,3]=0,
a[11,4]=-5137868354683248477586346693453125/495465427355479410622167542061972,
a[11,5]=-405677779726268282752892649892020224/71142722483104145892095323206788025,
a[11,6]=245921960510918251629683797176094768/59908811922681865929060892977735375,
a[11,7]=367953427734636456312437471451631/44380048256994921706877045210625,
a[11,8]=808591598711928461147452272/133856729103497890946741285,
a[11,9]=-3049240467/3093372235,
a[11,10]=3687441170432/6749831213125,
a[12,1]=18009539/351697500,
a[12,2]=0,
a[12,3]=0,
a[12,4]=0,
a[12,5]=0,
a[12,6]=1351744/5131875,
a[12,7]=898538649982788221/3655523545713900000,
a[12,8]=986955378624/4938547627475,
a[12,9]=397569776903/3090028048800,
a[12,10]=2957049856/29666896875,
a[12,11]=71669/6284880,

b[1]=18009539/351697500,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,
b[6]=1351744/5131875,
b[7]=898538649982788221/3655523545713900000,
b[8]=986955378624/4938547627475,
b[9]=397569776903/3090028048800,
b[10]=2957049856/29666896875,
b[11]=71669/6284880,

b*[1]=9132998184216727/177194863388812500,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=0,
b*[6]=5971284900143728/23270216609390625,
b*[7]=46560109562892125961981671/190526192880008411317500000,
b*[8]=11781220404761346096/57864557705069066875,
b*[9]=222323369516196533611/1556841029506080780000,
b*[10]=1246959894769246208/14946997743616640625,
b*[11]=-5978523233/2539290474000,
b*[12]=1/50.