An alternative "most efficient" 10 stage combined order 6 and 7 Runge-Kutta scheme

The following scheme is a minor variation of Jim Verner's ("most efficient") scheme from his website: http://www.math.sfu.ca/~jverner/. See: J.H. Verner, SIAM Journal of Numerical Analysis 1978, 772-790, "Explicit Runge-Kutta methods with estimates of the Local Truncation Error."

The nodes of the scheme are:

$$c_2 = \frac{1}{252},\ c_3 = \frac{86}{789},\ c_4 = \frac{43}{263},\ c_5 = \frac{57}{125},\ c_6 = \frac{13008053}{21291686},\ c_7 = \frac{61}{69},\ c_8 = \frac{13}{14},\ c_9 = 1,\ c_{10} = 1.$$

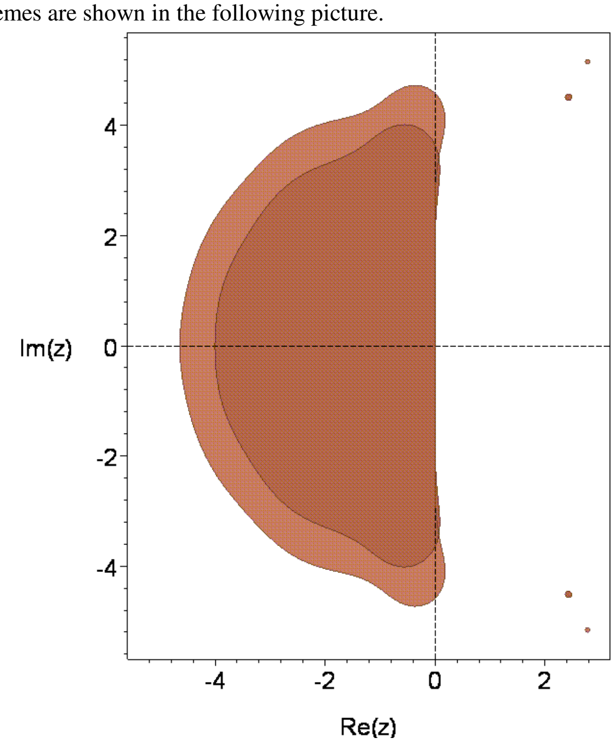
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1670628883 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.3712468252 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 186.7051158.

The 2-norm of the linking coefficients is: 265.7174228.

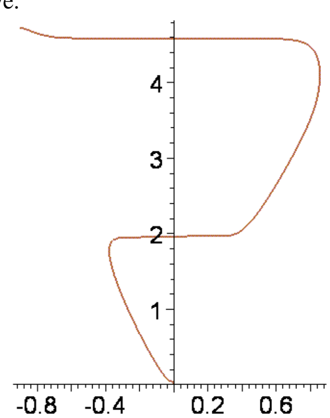
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6408, 0]$ and $[-4.0004, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[1.9601, 4.5850]$.

The coefficients in exact form are:
c[2]=1/252,
c[3]=86/789,
c[4]=43/263,
c[5]=57/125,
c[6]=13008053/21291686,
c[7]=61/69,
c[8]=13/14,
c[9]=1,
c[10]=1,

a[2,1]=1/252,
a[3,1]=-288014/207507,
a[3,2]=103544/69169,
a[4,1]=43/1052,
a[4,2]=0,
a[4,3]=129/1052,
a[5,1]=9241953609/14445312500,
a[5,2]=0,
a[5,3]=-35521879077/14445312500,
a[5,4]=8216746992/3611328125,
a[6,1]=-234661001374270154760381932343418837/86638708399372381264826530186047552,
a[6,2]=0,
a[6,3]=16708047681800687430447668443677501/1519977340339866337979412810281536,
a[6,4]=-62144311456998517127045299377825762331/7308051052354077353005016791833625088,
a[6,5]=187144289708101771545240859748046875/225288756075815256420381804832080384,
a[7,1]=156433223618472360600004453/25589069682685620798332196,
a[7,2]=0,
a[7,3]=-286228228632278279/11503926248984892,
a[7,4]=1602351681797903349299350451311/77954386526116243837729472574,
a[7,5]=-250101682295685807412109375/131840217089540049711772782,
a[7,6]=186968283848812602497818209919281152128/188152339243870737167151145420772913897,
a[8,1]=2416573506511307243928901879/189405179375617250759244608,
a[8,2]=0,
a[8,3]=-26845056973091253/509700334134208,
a[8,4]=6240574885620174185936288811795972315/143352880796432464872632124551324288,
a[8,5]=-653588044646464908208166015625/138571400963430037102110830912,
a[8,6]=1414671554060302249437031799034644611696958524689/66356240645416843519029683532987919994628608531,
a[8,7]=-784743211634853538227087/7124805416389431693881728,
a[9,1]=768824218294268200237382243/71169815816859510111759996,
a[9,2]=0,
a[9,3]=-335105412576515391/7383551972026652,
a[9,4]=216229112307376814409690903754219607/5688222216088376185796628168532102,
a[9,5]=-98929887953782580139019244140625/21530450284931693701095618902394,
a[9,6]=1618243877590754920440184413511290712322527881878400/767668869468652523201628063241559112654788951372623,

a[9,7]=4221295397605181483816184/14561567981373564250080527,
a[9,8]=-9294066721836866844621824/39758145220379691659171657,
a[10,1]=-299016622420082315893049777/6656990673921669938511852,
a[10,2]=0,
a[10,3]=1676281735372200609/8978231413171612,
a[10,4]=-3390261330726892468324760289843385/22098239472359586715309732752374,
a[10,5]=579836729004989073922626953125/31657240974212144460834535182,
a[10,6]=-45718947284177751978503039517324763532965011840/6546557672211342119280674815988605411648435757,
a[10,7]=1907389434109683715666805880/1469642468607140884094095421,
a[10,8]=0,
a[10,9]=0,

b[1]=293512833743/6217133891085,
b[2]=0,
b[3]=0,
b[4]=865525348711460452163965 7/33582639297730171365319680,
b[5]=14813090258407592773 4375/56315094351592106750054 4,
b[6]=611988202246692509405464414042481903274128359 6/4025589351034248352942783524429783913566182434 5,
b[7]=7405380323339247/15732275385648640,
b[8]=-492178523702404/1792132830157995,
b[9]=998316924287/11896622369280,
b[10]=0,

b*[1]=21386972732/478241068545,
b*[2]=0,
b*[3]=0,
b*[4]=4777499628943473191146 7/1788212955150701350656 00,
b*[5]=15123812915649414062 5/680956400865684483072,
b*[6]=1455170043817829351021486872797154011419 68/6700870334179899711249944586021048015648 45,
b*[7]=7080187535227393 11/3099258250972782080,
b*[8]=0,
b*[9]=0,
b*[10]=1213930694047/59483111846400.

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