An "efficient" 10 stage combined order 6 and 7 Runge-Kutta scheme

See: J.H. Verner, SIAM Journal of Numerical Analysis 1978, 772-790, "Explicit Runge-Kutta methods with estimates of the Local Truncation Error."

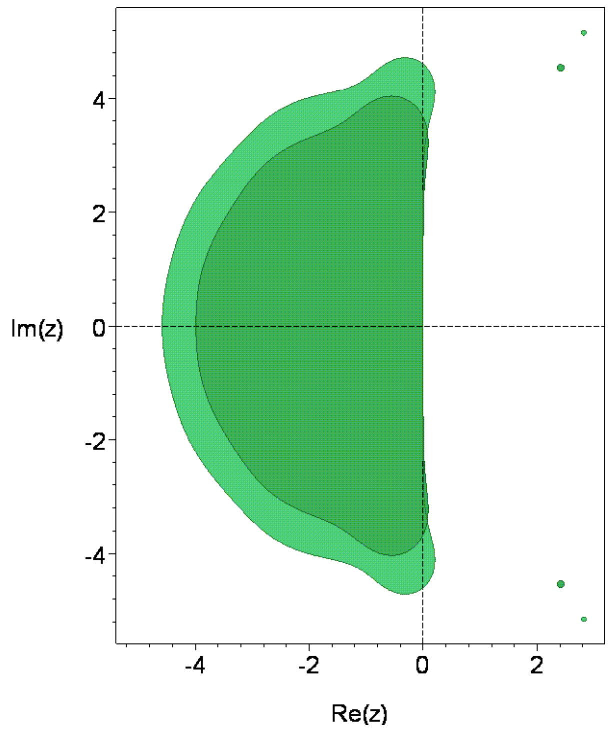
The nodes of the scheme are:

$$c_2 = \frac{1}{240},\ c_3 = \frac{4}{37},\ c_4 = \frac{6}{37},\ c_5 = \frac{4}{9},\ c_6 = \frac{5298}{9659},\ c_7 = \frac{37}{44},\ c_8 = \frac{11}{12},\ c_9 = 1,\ c_{10} = 1.$$

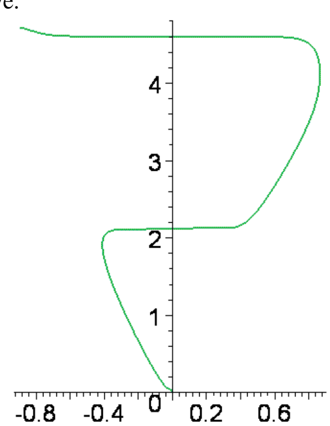
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2043042248 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.3360915091 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 31.87507758.
The 2-norm of the linking coefficients is: 57.22651913.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.
The real stability intervals of the order 7 and 6 schemes are respectively $[-4.5794, 0]$ and $[-3.9873, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[2.1163, 4.6026]$.

The coefficients in exact form are:
c[2]=1/240,
c[3]=4/37,
c[4]=6/37,
c[5]=4/9,
c[6]=5298/9659,
c[7]=37/44,
c[8]=11/12,
c[9]=1,
c[10]=1,

a[2,1]=1/240,
a[3,1]=-1772/1369,
a[3,2]=1920/1369,
a[4,1]=3/74,
a[4,2]=0,
a[4,3]=9/74,
a[5,1]=3878/6561,
a[5,2]=0,
a[5,3]=-4958/2187,
a[5,4]=13912/6561,
a[6,1]=-16360887763789593/17408392096861922,
a[6,2]=0,
a[6,3]=68007651756649533/17408392096861922,
a[6,4]=-116357224700105584/409097214276255167,
a[6,5]=174264745828087332/409097214276255167,
a[7,1]=10375359957812891/10219736543182848,
a[7,2]=0,
a[7,3]=-490218990633/107165560832,
a[7,4]=276746734415529506O5/6258946282327867392,
a[7,5]=-125124504250240755/91125448348909568,
a[7,6]=71883621007388690573300 5/531851829635693143425024,
a[8,1]=16935236372154617/2866775355703296,
a[8,2]=0,
a[8,3]=-6299404771/197627904,
a[8,4]=2440834998651506195/772987696788602 88,
a[8,5]=-545466290060829555/361551621787320 32,
a[8,6]=153764430955779515440610 5285/1366599502889075829063 35232,
a[8,7]=-409808923530490/478134615228543,
a[9,1]=7231408665258653/5936112042607296,
a[9,2]=0,
a[9,3]=-309529827/38235188,
a[9,4]=5962321806871382776391/661311819888255169920,
a[9,5]=-104753172313981755/20417723144572736,
a[9,6]=22771074337949893498709586 14245/5750208159210298860655981 62816,
a[9,7]=112959716373203392/108927778388930 6245,
a[9,8]=-108122439345984/1503120876043985,
a[10,1]=-300758382639/103988917952,
a[10,2]=0,
a[10,3]=6329479/397864,
a[10,4]=-52700259792454777/3423589157678976,
a[10,5]=866081714994585/106230378691904,
a[10,6]=-1333940883992770281096 05/233695610484589762248 96,
a[10,7]=3169210479408512/3400410241550283,
a[10,8]=0,
a[10,9]=0,

b[1]=1014533507/21735362880,
b[2]=0,
b[3]=0,
b[4]=96242498931518971/375320934516528000,
b[5]=75572268327/373802430400,
b[6]=283412123024313086748058309/1567648323260008495806706560,
b[7]=7293955984128/23833382311375,
b[8]=-1371043584/27518750875,
b[9]=9558797/165608975,
b[10]=0,

b*[1]=2565083/56455488,
b*[2]=0,
b*[3]=0,
b*[4]=41839300551563/160051571222400,
b*[5]=9308700873/53400347200,
b*[6]=6641832540911777531621/30738540526108296900480,
b*[7]=147352646144/569777652925,
b*[8]=0,
b*[9]=0,
b*[10]=149199/3379775.

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