A "robust" 10 stage combined order 6 and 7 Runge-Kutta scheme

See: J.H. Verner, SIAM Journal of Numerical Analysis 1978, 772-790, "Explicit Runge-Kutta methods with estimates of the Local Truncation Error."

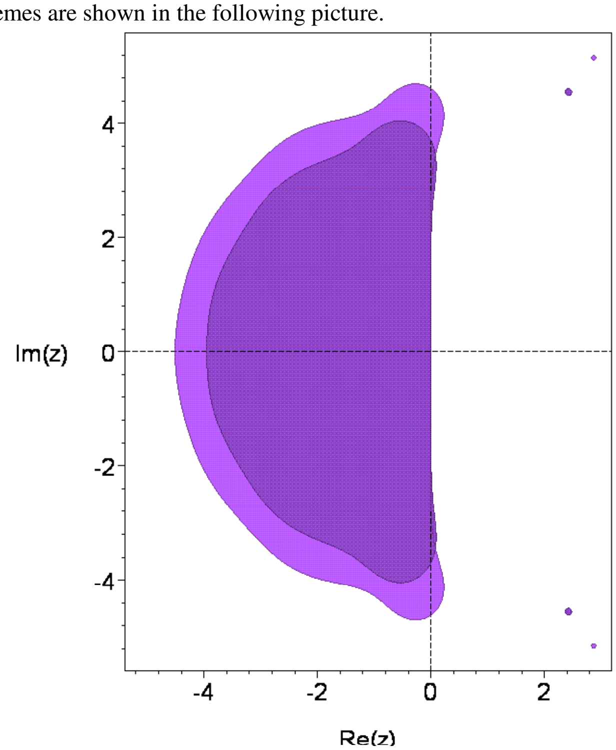
The nodes of the scheme are:

$$c_2 = \frac{1}{250},\ c_3 = \frac{28}{255},\ c_4 = \frac{14}{85},\ c_5 = \frac{11}{25},\ c_6 = \frac{37771}{77681},\ c_7 = \frac{4}{5},\ c_8 = \frac{9}{10},\ c_9 = 1,\ c_{10} = 1.$$

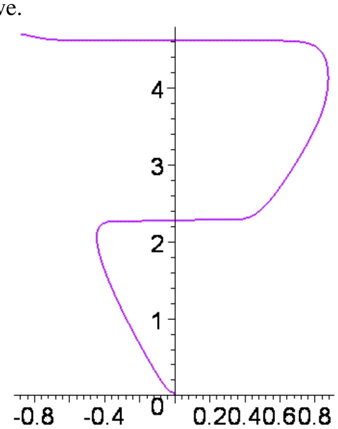
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2409311094 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.3507418686 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 20.30040051.
The 2-norm of the linking coefficients is: 44.89284041.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.
The real stability intervals of the order 7 and 6 schemes are respectively $[-4.5116, 0]$ and $[-3.9519, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[2.2775, 4.6162]$.

The coefficients in exact form are:
c[2]=1/250,
c[3]=28/255,
c[4]=14/85,
c[5]=11/25,
c[6]=37771/77681,
c[7]=4/5,
c[8]=9/10,
c[9]=1,
c[10]=1,

a[2,1]=1/250,
a[3,1]=-18172/13005,
a[3,2]=3920/2601,
a[4,1]=7/170,
a[4,2]=0,
a[4,3]=21/170,
a[5,1]=33121/61250,
a[5,2]=0,
a[5,3]=-253011/122500,
a[5,4]=240669/122500,
a[6,1]=4222494378009329586083/39253464326509271584438,
a[6,2]=0,
a[6,3]=-2252824606828982945235/7136993513910776651716,
a[6,4]=51439540868114936232105/92780915680840096472308,
a[6,5]=728458292292718545000/5207092206577760516303,
a[7,1]=-5299193429587/5954372468275,
a[7,2]=0,
a[7,3]=8866707/2866255,
a[7,4]=-2952750917616/1757906866807,
a[7,5]=-11072596875/4172354758,
a[7,6]=19042169027464066143/6499072341594230350,
a[8,1]=122674005636625593/20883175120734080,
a[8,2]=0,
a[8,3]=-1582768017/91720160,
a[8,4]=17718995066535794425/1872719280091898784,
a[8,5]=120309906551678125/5926479455939328,
a[8,6]=-30772472672589976565694954455/1666696032247163511057539328,
a[8,7]=56738488975/57701559168,
a[9,1]=-24597779751957752/9374959112800743,
a[9,2]=0,
a[9,3]=1921446335/271040364,
a[9,4]=-185090168340709143079/67256694747280541412,
a[9,5]=-6795405176638474375/611923978037124324,
a[9,6]=3574124121615579418693597598118415/343560130499414026035721185687396,
a[9,7]=-397390460637625/3030464084895522,
a[9,8]=58132547384704/486321506408277,
a[10,1]=655291200027306/195423104986571,
a[10,2]=0,
a[10,3]=-17994223665/1881415564,
a[10,4]=21058354131888743695/4205942221348304892,
a[10,5]=7897813353311875/554595271180236,
a[10,6]=-18292306674856394863199745865/1403712896275231293237797124,
a[10,7]=188542919280625/189512090845902,
a[10,8]=0,
a[10,9]=0,

b[1]=277826609/5863267872,
b[2]=0,
b[3]=0,
b[4]=97517058051157/373313964690576,
b[5]=226462890625/9376352137152,
b[6]=10261378958310617950057158665l827/315146279193663034614455208169920,
b[7]=1595451125/6633573408,b[8]=25644208/465736131,
b[9]=51165783/1110775120,
b[10]=0,

b*[1]=4664353/93067744,
b*[2]=0,
b*[3]=0,
b*[4]=4426936643875/17776855461456,
b*[5]=3061328125/19412737344,
b*[6]=282861629160256653613240l/1556324265822152800967l360,
b*[7]=95054375/315884448,
b*[8]=0,
b*[9]=0,
b*[10]=9599059/158682160.

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