

Verner's "most robust" 10 stage combined order 6 and 7 Runge-Kutta scheme

The following ("most robust") scheme comes from Jim Verner's website: <http://www.math.sfu.ca/~jverner/>.

See: J.H. Verner, SIAM Journal of Numerical Analysis 1978, 772-790, "Explicit Runge-Kutta methods with estimates of the Local Truncation Error."

The nodes of the scheme are:

$$c_2 = \frac{1}{200}, c_3 = \frac{49}{450}, c_4 = \frac{49}{300}, c_5 = \frac{91}{200}, c_6 = \frac{34704460}{57271701}, c_7 = \frac{167}{200}, c_8 = \frac{183}{200}, c_9 = 1, c_{10} = 1.$$

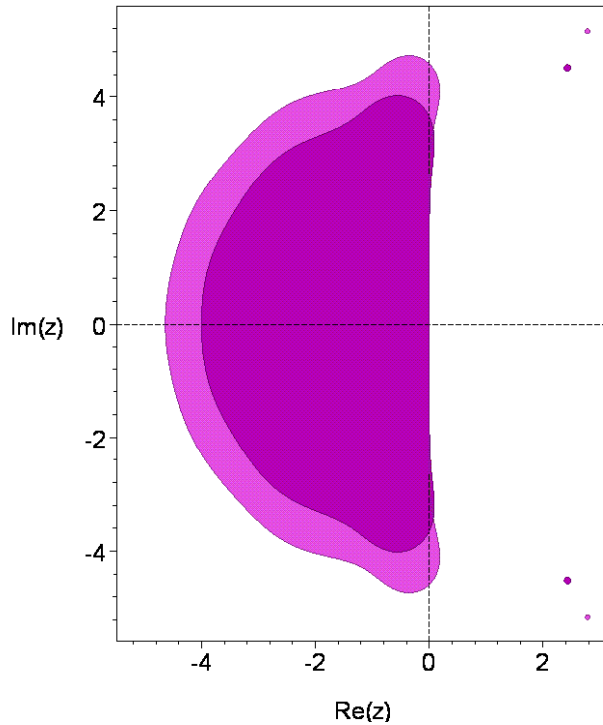
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2701546765 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.3333558768 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 80.49553671.

The 2-norm of the linking coefficients is: 119.7099807.

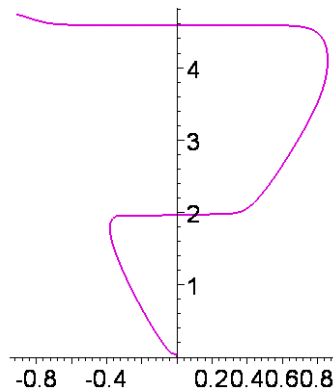
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6355, 0]$ and $[-3.9995, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[1.9740, 4.5865]$.

The coefficients in exact form are:

- c[2]=1/200,
- c[3]=49/450,
- c[4]=49/300,
- c[5]=91/200,
- c[6]=34704460/57271701,
- c[7]=167/200,
- c[8]=183/200,
- c[9]=1,
- c[10]=1,

- a[2,1]=1/200,
- a[3,1]=-4361/4050,
- a[3,2]=2401/2025,
- a[4,1]=49/1200,
- a[4,2]=0,
- a[4,3]=49/400,
- a[5,1]=1781/2800,
- a[5,2]=0,
- a[5,3]=-13689/5600,
- a[5,4]=507/224,
- a[6,1]=-275776923568321554889485313326460/108782544039075797415588764982099,
- a[6,2]=0,
- a[6,3]=9576001512158705648097438730000/929765333667314507825544999847,
- a[6,4]=-22178538465642954902290458689600/2789296001001943523476634999541,
- a[6,5]=12323686750109283453854913779200/15540363434153685345084109283157,
- a[7,1]=8221508014471276464993/820610858494509040000,
- a[7,2]=0,
- a[7,3]=-9852144759099/2364568872400,
- a[7,4]=15322550932778398907299/3996134347464283007200,
- a[7,5]=-18338463121898520004/36506562121215938675,
- a[7,6]=23340475544602125119307511373519383499/34957329425893779598660175543672800000,
- a[8,1]=81088643022740545033730780169/2975182110231937140152800000,
- a[8,2]=0,
- a[8,3]=-2837794586103/67559110640,
- a[8,4]=-14167575606881316095038341141/134471918859346855337836000,
- a[8,5]=2395552232834276839307772/29760062864388068268875,
- a[8,6]=-4076715891031001341580357765362043260356514682697/
60535801523558513633981092635987721507186400000,
- a[8,7]=36551527355459957808/2801171464968864455,
- a[9,1]=-3347747115771808477876752833599/1101327591307901549464211073280,
- a[9,2]=0,
- a[9,3]=1214704878477625125/119815105452943264,
- a[9,4]=-65581118617864038124456452415/10200342297342072539428709672,
- a[9,5]=-133373082911575479273298406095/84070826916189821955373830704,
- a[9,6]=622515683654039386383701463758952267447736841050281950137693/
328994218860140584540186142455568117669077094653332432085760,
- a[9,7]=46169188671551441399013763125/2343692704156841275628930358208,
- a[9,8]=18880867865877597493091875/3469664148196911643475533504,
- a[10,1]=-74309815528722196127526037/51427190037752065777334640,
- a[10,2]=0,
- a[10,3]=913722369253875/113761793498234,
- a[10,4]=-440658227159292602060396890/58109996881684093545238767,
- a[10,5]=37290888293935763341814380/10411746696360295961914239,
- a[10,6]=-645152888113581065780360392105179310452496326847/264735425121804814898131042131367320451487910960,
- a[10,7]=473757486190086838681892500/556321100802942639360459887,
- a[10,8]=0,
- a[10,9]=0,

b[1]=9420080774669597/198627609019792680,
b[2]=0,
b[3]=0,
b[4]=18658605936510000/72821569629535727,
b[5]=296950875175030000/1101802245630054969,
b[6]=18875276980274212686824835566143151189850553896330009/
148780947139609706104394157596648357994575577036224440,
b[7]=18663850606812500/74993696164706319,
b[8]=179884749312500/58508928482581269,
b[9]=349315176247648/7273791403140339,
b[10]=0,

b*[1]=7362904929137/155056681514280,
b*[2]=0,
b*[3]=0,
b*[4]=7505178129270000/29317774785916981,
b*[5]=1851744839320000/6843492208882329,
b*[6]=750882778189818437512810407839645051128089/6004363295715536270735789992319176322209240,
b*[7]=27902602073000000/110704980052661709,
b*[8]=0,
b*[9]=0,
b*[10]=331667036438/6791588611709.

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