A 7 stage, order 6 Runge-Kutta scheme with an 8 stage, order 5 embedded scheme

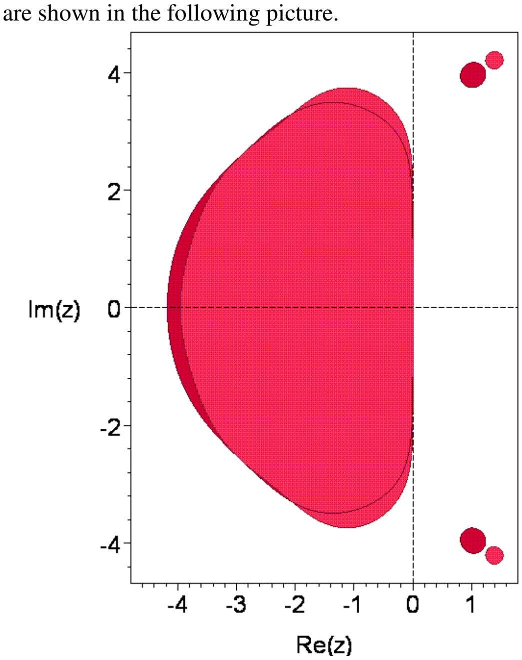
See: On the Optimization of Some Seven-stage Sixth-order Runge-Kutta Methods, by M. Tanaka, K. Kasuga, S. Yamashita and H. Yazaki, Journal of the Information Processing Society of Japan, Vol. 33, No. 8 (1992), pages 993 to 1005.

The order 6 scheme considered here is a minor variation of a scheme given in the preceding paper.
The values for c_3, c_5 and c_6 are chosen to minimize the principal error norm.
The nodes of the scheme are:

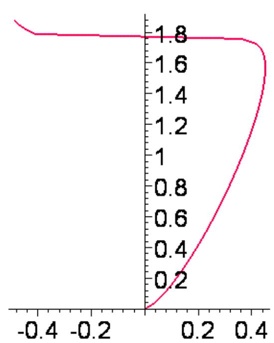
$$c_2 = \frac{30}{167},\ c_3 = \frac{2}{9},\ c_4 = \frac{3}{7},\ c_5 = \frac{140}{201},\ c_6 = \frac{227}{293},\ c_7 = 1,\ c_8 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is: $0.2106258537 \times 10^{(-3)}$.
Note: Two of the 48 principal error conditions are satisfied by the order 6 scheme.
The principal error norm of the order 5 embedded scheme is: $0.7714522089 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: $\frac{1778829409}{519886770} \simeq 3.421570834$.
The 2-norm of the linking coefficients is: 5.328769422.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-3.9541, 0]$ and $[-4.1774, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 1.7644]$.

The coefficients are:
c[2]=30/167,
c[3]=2/9,
c[4]=3/7,
c[5]=140/201,
c[6]=227/293,
c[7]=1,
c[8]=1,

a[2,1]=30/167,
a[3,1]=103/1215,
a[3,2]=167/1215,
a[4,1]=681/6860,
a[4,2]=-1503/3430,
a[4,3]=1053/1372,
a[5,1]=1875916/8120601,
a[5,2]=-488642/2706867,
a[5,3]=2730/300763,
a[5,4]=5172440/8120601,
a[6,1]=-234878478619063/2928268584253320,
a[6,2]=1244059653/2766913270,
a[6,3]=103872061332063/589139750879537,
a[6,4]=-3949178196144055/51255158326519719,
a[6,5]=2928389813501681 61/956762955428368088,
a[7,1]=392156636/1721693985,
a[7,2]=-2641773/5056370,
a[7,3]=234673827885/498018061684,
a[7,4]=1385083606718/2639205187905,
a[7,5]=-95838799749201/228334615975180,
a[7,6]=415405543347564/578242061880635,
a[8,1]=-442047853/386122800,
a[8,2]=235637/229835,
a[8,3]=69586947/23902840,
a[8,4]=-1778829409/519886770,
a[8,5]=79088337717/48522765200,
a[8,6]=0,
a[8,7]=0,

b[1]=832301/11440800,
b[2]=0,
b[3]=39674367/137890480,
b[4]=16353211/86392800,
b[5]=36453377889/265963952800,
b[6]=92855270041799/389461444858800,
b[7]=45967/614880,
b[8]=0,

b*[1]=260431/4086000,
b*[2]=0,
b*[3]=2076192/6155825,
b*[4]=283392431/3131739000,
b*[5]=16141047921/45157814000,
b*[6]=19818066603889/324551204049000,
b*[7]=1/25,
b*[8]=1/20.

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