An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

This scheme is constructed to have a stability polynomial suggested by Ch. Tsitouras and consequently has a large stability region.

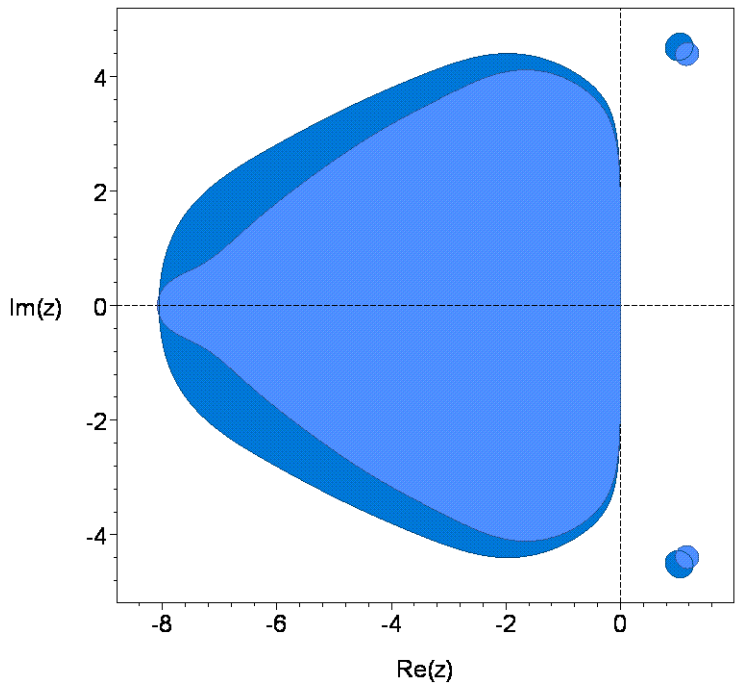
See: A Parameter Study of Explicit Runge-Kutta Pairs of Orders 6(5), by Ch. Tsitouras,
Applied Mathematics Letters, Vol. 11, No. 1, pages 65 to 69, 1998.

The nodes of the scheme are:

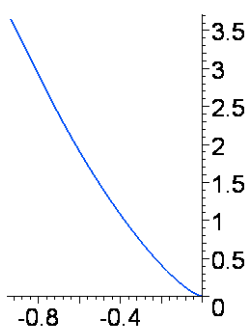
$$c_2 = \frac{1}{192},\ c_3 = \frac{56}{435},\ c_4 = \frac{28}{145},\ c_5 = \frac{56}{169},\ c_6 = \frac{37}{58},\ c_7 = \frac{121}{158},\ c_8 = 1,\ c_9 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is: $0.2936111560 \times 10^{(-3)}$.
The principal error norm of the order 5 embedded scheme is: $0.2216515665 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 2.623768968.
The 2-norm of the linking coefficients is: 5.596229081.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-8.0893, 0]$ and $[-8.0581, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis only at the origin.

The coefficients are:
c[2]=1/192,
c[3]=56/435,
c[4]=28/145,
c[5]=56/169,
c[6]=37/58,
c[7]=121/158,
c[8]=1,
c[9]=1,

a[2,1]=1/192,
a[3,1]=-30744/21025,
a[3,2]=100352/63075,
a[4,1]=7/145,
a[4,2]=0,
a[4,3]=21/145,
a[5,1]=523516/4826809,
a[5,2]=0,
a[5,3]=-889140/4826809,
a[5,4]=1965040/4826809,
a[6,1]=41044746763290003/51739779980736512,
a[6,2]=0,
a[6,3]=-57685603608510645/24977824818286592,
a[6,4]=83145940696701395/53969942910940672,
a[6,5]=53772835209542058935/87647187287367651328,
a[7,1]=-197373717285144929204576 23/384824669721407219242 37312,
a[7,2]=0,
a[7,3]=27466253014073155807766505/1456093344891811099836 0064,
a[7,4]=-11767013410215863112783 8015/12298788430961190182 5434112,
a[7,5]=-19483500956399449871589744537/48131165515398815905079 1915520,
a[7,6]=913336658253963141/2343933323194077385,
a[8,1]=24848139541710316158 07/2538960546022287823744,
a[8,2]=0,
a[8,3]=-22914752451723638749 5/87335252197326853888,
a[8,4]=13054841587550170871 6128185/10646878869203744207 5798624,
a[8,5]=11103534575830173429908962617/81865767619288881165 08540672,
a[8,6]=-10679027011796052/13181450824860817,
a[8,7]=223837786904659964 4/2564667687663709133,
a[9,1]=397933/5470080,
a[9,2]=0,
a[9,3]=0,
a[9,4]=5450042155625/32205284983872,
a[9,5]=1677324270326783/6406236767145600,
a[9,6]=179200957846/1323774725175,
a[9,7]=77461817788426/272279942939565,
a[9,8]=47049059/616363020,

b[1]=397933/5470080,
b[2]=0,
b[3]=0,
b[4]=5450042155625/32205284983872,
b[5]=1677324270326783/6406236767145600,
b[6]=179200957846/1323774725175,
b[7]=77461817788426/272279942939565,
b[8]=47049059/616363020,

b*[1]=58931524118239849/837231220422643200,
b*[2]=0,
b*[3]=0,
b*[4]=1800771389181962432052 25/98584554708911678024 2176,
b*[5]=23900627890862505647859 8819/98051608511643687347 4624000,
b*[6]=4448038142979447869029/2894461529742355163 8500,
b*[7]=1616804268713313415249069/5953458735623079359484300,
b*[8]=7135366796284370 51/13476908236860104400,
b*[9]=1/40.

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