An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

The scheme considered here has $c_6 = \frac{29}{39}$ and $c_7 = \frac{575}{576}$.

With c_6 and c_7 having these fixed values, the nodes c_2, c_4 and c_5 have been chosen to minimize the principal error norm.

The nodes of the scheme are:

$$c_2 = \frac{21}{229},\ c_3 = \frac{398}{2805},\ c_4 = \frac{199}{935},\ c_5 = \frac{541}{959},\ c_6 = \frac{29}{39},\ c_7 = \frac{575}{576},\ c_8 = 1,\ c_9 = 1.$$

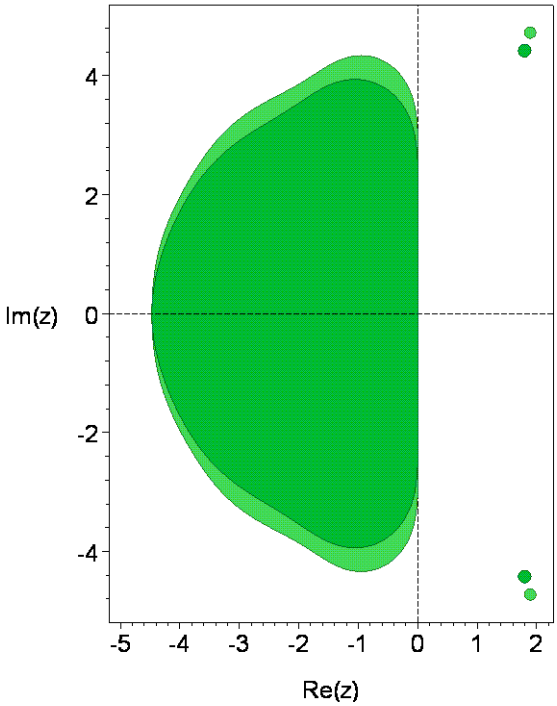
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1037547445 \times 10^{(-4)}$.

The principal error norm of the order 5 embedded scheme is: $0.6303816622 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 32.86795411.

The 2-norm of the linking coefficients is: 62.89536207.

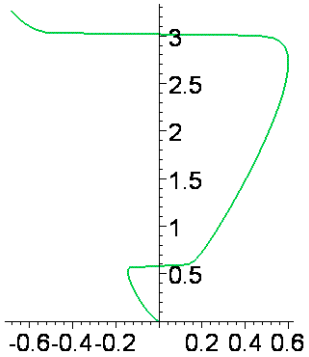
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively $[-4.4717, 0]$ and $[-4.4717, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: [0.5862, 3.0103].

The coefficients are:
c[2]=21/229,
c[3]=398/2805,
c[4]=199/935,
c[5]=541/959,
c[6]=29/39,
c[7]=575/576,
c[8]=1,
c[9]=1,

a[2,1]=21/229,
a[3,1]=5306932/165228525,
a[3,2]=18137258/165228525,
a[4,1]=199/3740,
a[4,2]=0,
a[4,3]=597/3740,
a[5,1]=94539185952859/139708222009916,
a[5,2]=0,
a[5,3]=-360526602615135/139708222009916,
a[5,4]=86200229584590/34927055502479,
a[6,1]=-897896928848000024385758708 9/309478221633141626395690 2924,
a[6,2]=0,
a[6,3]=99268298611547543700685925/826292232744062670598 4748,
a[6,4]=-16240947264530777128270036 3150/173261380181127920988347 90979,
a[6,5]=40197004738132352835791693452/400018048245760073938629353 91,
a[7,1]=20286604731599462128958430510992102863475/25821252226156041404085984294142098800 64,
a[7,2]=0,
a[7,3]=-75963452911125085054379188438047598 75/23772939210064272792198199433704243 2,
a[7,4]=66207885165898300271256280071036892800616153 75/25083900175630865229822206008706748964562534 4,
a[7,5]=-16378468176189294528981106978962032922592625/772468438059969385382672071798267914800332 8,
a[7,6]=97768503635786059874525/119061117135805961207808,
a[8,1]=13855115178421131108323299597371007/171512201558252472735999614929062 0,
a[8,2]=0,
a[8,3]=-23954172438450319856818854832 5/72880022758303045758599279 72,
a[8,4]=95651830919667211530668834468508100881883600/352390493508218844863451281054550483347915 1,
a[8,5]=-74169047208676368580733278643243477207 2/33803569324846358753294490754564762371 9,
a[8,6]=257132510132640765480/305396035838269868597,
a[8,7]=-11870598998037280849 92/65786922727385059942689 5,
a[9,1]=1697710672/26928207375,
a[9,2]=0,
a[9,3]=0,
a[9,4]=16449003836325341187 5/503598049753427901952,
a[9,5]=68826329186331197868 1/2293101349345635227520,
a[9,6]=237419997287733/1436816713308800,
a[9,7]=21298921647196078 08/556154447061193625,
a[9,8]=-680161433/184588800,

b[1]=1697710672/26928207375,
b[2]=0,
b[3]=0,
b[4]=16449003836325341187 5/503598049753427901952,
b[5]=68826329186331197868 1/2293101349345635227520,
b[6]=237419997287733/1436816713308800,
b[7]=21298921647196078 08/556154447061193625,
b[8]=-680161433/184588800,

b*[1]=1097924377519127782369/1797442392254582442600 0,
b*[2]=0,
b*[3]=0,
b*[4]=22948656626877224955778831923125/68757710895541194720834617432064,
b*[5]=64226939084201757950818485359 41/2342804072520515602009637336832 0,
b*[6]=54099781942778719068439 31/26640747567082588647769600,
b*[7]=68756994059899966271143280 64/25779854339158373920403533 75,
b*[8]=-1117082086684662594 7/4400427318713395200,
b*[9]=-1/1764.

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