

## A modification of Papakostas' 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

The scheme considered here has  $c_6 = \frac{73}{99}$  and  $c_7 = \frac{199}{200}$ .

With  $c_6$  and  $c_7$  having these fixed values, the nodes  $c_2$ ,  $c_4$  and  $c_5$  have been chosen to minimize the principal error norm.

The nodes of the scheme are:

$$c_2 = \frac{1}{11}, c_3 = \frac{157}{1104}, c_4 = \frac{157}{736}, c_5 = \frac{123}{218}, c_6 = \frac{73}{99}, c_7 = \frac{199}{200}, c_8 = 1, c_9 = 1.$$

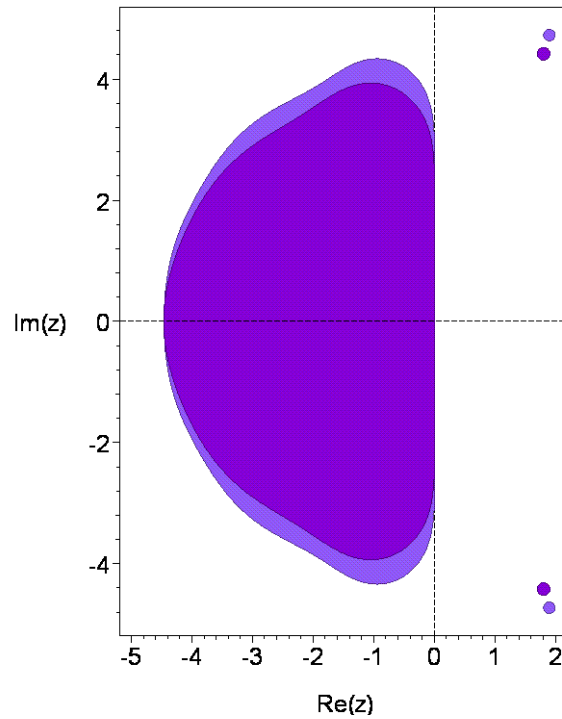
The principal error norm, that is, the 2-norm of the principal error terms is:  $0.1128941603 \times 10^{(-4)}$ .

The principal error norm of the order 5 embedded scheme is:  $0.6199568809 \times 10^{(-3)}$ .

The maximum magnitude of the linking coefficients is: 30.34060818.

The 2-norm of the linking coefficients is: 56.61131252.

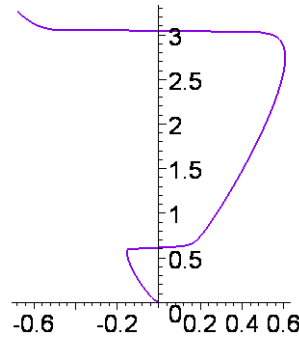
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively  $[-4.4595, 0]$  and  $[-4.4639, 0]$ .

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval:  $[0.6275, 3.0415]$ .

The coefficients are:

$c[2]=1/11$ ,  
 $c[3]=157/1104$ ,  
 $c[4]=157/736$ ,  
 $c[5]=123/218$ ,  
 $c[6]=73/99$ ,  
 $c[7]=199/200$ ,  
 $c[8]=1$ ,  
 $c[9]=1$ ,

$a[2,1]=1/11$ ,  
 $a[3,1]=75517/2437632$ ,  
 $a[3,2]=271139/2437632$ ,  
 $a[4,1]=157/2944$ ,  
 $a[4,2]=0$ ,  
 $a[4,3]=471/2944$ ,  
 $a[5,1]=42928977471/63842339642$ ,  
 $a[5,2]=0$ ,  
 $a[5,3]=-81818872578/31921169821$ ,  
 $a[5,4]=78364952136/31921169821$ ,  
 $a[6,1]=-452993729346115584620105837/169351895877446242624608351$ ,  
 $a[6,2]=0$ ,  
 $a[6,3]=1697439018580941844337032/152982742436717473012293$ ,  
 $a[6,4]=-334578912430118682410335643360/38759554641024302244921542187$ ,  
 $a[6,5]=183506358169025606635433230/193412520623391990592938849$ ,  
 $a[7,1]=237424966244801792240477643719598303/34580776953279718763722454600000000$ ,  
 $a[7,2]=0$ ,  
 $a[7,3]=-20179674002037028690247750335371/722117794714327571912012500000$ ,  
 $a[7,4]=128453834152286986408251909051997320271/5544554200138077926323128675315625000$ ,  
 $a[7,5]=-972708878592287689266824375699676121/505439891163443236912399043150000000$ ,  
 $a[7,6]=199423777107739967674947/239913433051724600000000$ ,  
 $a[8,1]=5973266207518682347729280873413/802324115518564191458832770081$ ,  
 $a[8,2]=0$ ,  
 $a[8,3]=-40870912569246870680146344/1347069654182311812082183$ ,  
 $a[8,4]=112459036656658044255344022793238112/4474108065454386923650087599365353$ ,  
 $a[8,5]=-4712736023245029582064053285795650/2213621528406610002679128666238199$ ,  
 $a[8,6]=127657765502309225898/142682894776218689669$ ,  
 $a[8,7]=-541736066625000000/96886654438663742381$ ,  
 $a[9,1]=531842063/8415926910$ ,  
 $a[9,2]=0$ ,  
 $a[9,3]=0$ ,  
 $a[9,4]=6900573846223978496/21081697499495907225$ ,  
 $a[9,5]=51126843079934806/173160464034159675$ ,  
 $a[9,6]=1147778420925807/6907762998459050$ ,  
 $a[9,7]=575779000000000/411330558898641$ ,  
 $a[9,8]=-13426037/10725975$ ,

$b[1]=531842063/8415926910$ ,  
 $b[2]=0$ ,  
 $b[3]=0$ ,  
 $b[4]=6900573846223978496/21081697499495907225$ ,  
 $b[5]=51126843079934806/173160464034159675$ ,  
 $b[6]=1147778420925807/6907762998459050$ ,  
 $b[7]=575779000000000/411330558898641$ ,  
 $b[8]=-13426037/10725975$ ,

$b^*[1]=648854567388972967/10596897420732888642,$   
 $b^*[2]=0,$   
 $b^*[3]=0,$   
 $b^*[4]=8879698038617778833446469632/26544976952167847097500780295,$   
 $b^*[5]=58500328551366236723248598/218034649578069682785496485,$   
 $b^*[6]=32934117770434118898396/161072145715862713160465,$   
 $b^*[7]=2584290081034720820000000/2589630224498721525333771,$   
 $b^*[8]=-4674424280471995/5402235928512618,$   
 $b^*[9]=-1/845.$

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