An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

The scheme considered here has $c_6 = \frac{64}{91}$ and $c_7 = \frac{63}{64}$.

With c_6 and c_7 having these fixed values, the nodes c_2, c_4 and c_5 have been chosen to minimize the principal error norm.

The nodes of the scheme are:

$$c_2 = \frac{19}{220},\ c_3 = \frac{248}{1719},\ c_4 = \frac{124}{573},\ c_5 = \frac{159}{281},\ c_6 = \frac{64}{91},\ c_7 = \frac{63}{64},\ c_8 = 1,\ c_9 = 1.$$

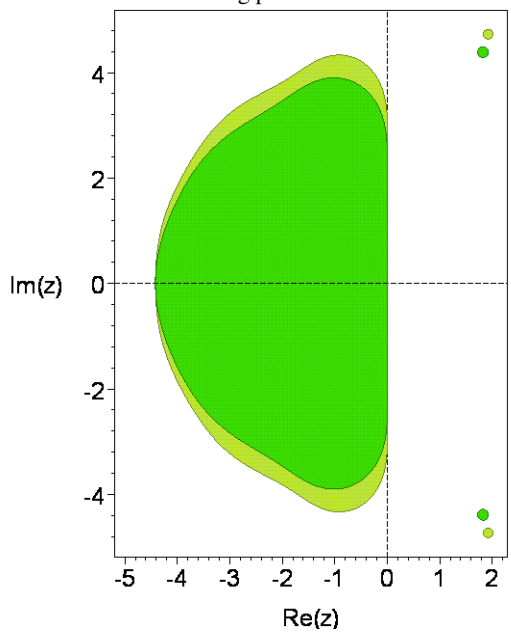
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1746831711 \times 10^{(-4)}$.

The principal error norm of the order 5 embedded scheme is: $0.5840380502 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 17.48095317.

The 2-norm of the linking coefficients is: 30.93117482.

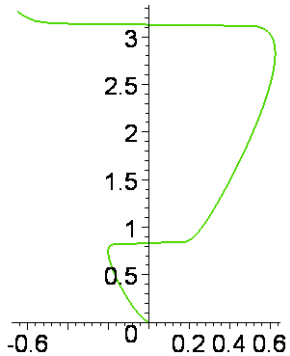
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively $[-4.4174, 0]$ and $[-4.4195, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: [0.8371, 3.1337].

The coefficients are:
c[2]=19/220,
c[3]=248/1719,
c[4]=124/573,
c[5]=159/281,
c[6]=64/91,
c[7]=63/64,
c[8]=1,
c[9]=1,

a[2,1]=19/220,
a[3,1]=1334488/56144259,
a[3,2]=6765440/56144259,
a[4,1]=31/573,
a[4,2]=0,
a[4,3]=31/191,
a[5,1]=443985772809/682326636832,
a[5,2]=0,
a[5,3]=-1687953692799/682326636832,
a[5,4]=815026549419/341163318416,
a[6,1]=-1777725098418521712533422 4/1151829473640302780970674 3,
a[6,2]=0,
a[6,3]=1412202709832277291195888/2173263157811892039567 31,
a[6,4]=-600267622176623241716824639 68/1222743050479704818221755 6253,
a[6,5]=4435917233994487983443891 20/6743536074445822618704791 69,
a[7,1]=3116283851584560023118564368757831/9553019281847695820720712680 73472,
a[7,2]=0,
a[7,3]=-1506935890711156475983912961 5089/1126535292670718846783102910464,
a[7,4]=4608655696349942937342097596091097426937/4022871735737177579627118428194 64495104,
a[7,5]=-1041097287013501701647909592093443697/7679368644108139871039490290509 41440,
a[7,6]=1267059613614654529943 91/1268854616095504675635 20,
a[8,1]=2688622152256014375593487443/6358269443841942299620775 52,
a[8,2]=0,
a[8,3]=-1650644306962913079626 49/9442530342150159719648,
a[8,4]=8564734413440506162620603830967146870 7/5745317881110570819440070776064368272,
a[8,5]=-1868050624284923482186332306552702/9770687856009822698317026814 78435,
a[8,6]=1684265320058783471 1/1316647497491741008 0,
a[8,7]=-608491123177947136/2736683559896951765 1,
a[9,1]=2465359/38465280,
a[9,2]=0,
a[9,3]=0,
a[9,4]=29999319743551167/90312244465616140,
a[9,5]=2440521938223593/9364974686865720,
a[9,6]=1241823968749/6822515208960,
a[9,7]=177732994465792/327930128445465,
a[9,8]=-11266189/29580120,

b[1]=2465359/38465280,
b[2]=0,
b[3]=0,
b[4]=29999319743551167/90312244465616140,
b[5]=2440521938223593/9364974686865720,
b[6]=1241823968749/6822515208960,
b[7]=177732994465792/327930128445465,
b[8]=-11266189/29580120,

b*[1]=739837987028857/11890322590648320,
b*[2]=0,
b*[3]=0,
b*[4]=120499294582674885160316209 5/3551063890812398485590445952,
b*[5]=328936061938716185672737/1439660654341730577416400,
b*[6]=102717532147008407320237/4618004088297246045696 00,
b*[7]=9268295603184461676544/2166358099367980513768 5,
b*[8]=-9633553238997677/35173968040963440,
b*[9]=-1/156.

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