

Tsitouras' (2009) 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

See: Runge–Kutta pairs of orders 5(4) using the minimal set of simplifying assumptions,
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The nodes of the scheme are:

$$c_2 = \frac{355}{1533}, c_3 = \frac{343}{1616}, c_4 = \frac{2743}{4597}, c_5 = \frac{2399}{3010}, c_6 = 1, c_7 = 1.$$

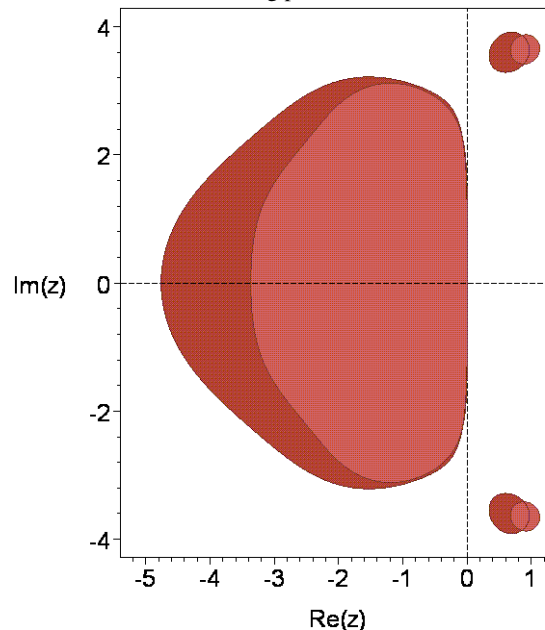
The principal error norm, that is, the 2-norm of the principal error terms is: $0.5232270032 \times 10^{(-3)}$.

The principal error norm of the order 4 embedded scheme is: $0.7612088861 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 9.516272353.

The 2-norm of the linking coefficients is: 14.70461404.

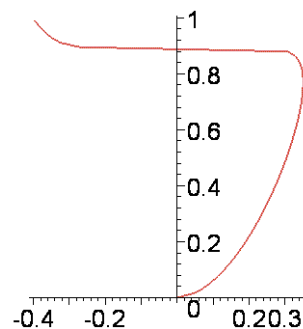
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.

The real stability intervals of the order 5 and 4 schemes are respectively $[-3.3670, 0]$ and $[-4.7631, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[0, 0.8923]$.

c[2]=.2315720808871493803000652315720808871493803000652315720808871493803000652315720808871,
c[3]=.212252475,
c[4]=.5966934957581031107243854687839895584076571677180770067435283880791820752664781379160,
c[5]=.7970099667774086378737541528239202657807308970099667774086378737541528239202657807309,
c[6]=1.,
c[7]=1.,

[a[2,1]=.231572080887149383000652315720808871493803000652315720808871493803000652315720808871,
a[3,1]=.2713559481460930140207448657181945340323682659628011716129384562063894144499879291471,
a[3,2]=-.5910347289856826154549734096571928650761579071527641913769093145391416692523545389956e-1,
a[4,1]=.4307300405630893663751393295225826273051600040980401794213433452487436784989886248844e-1,
a[4,2]=4.560099159915419678879073451407155744076889086367506920759386915218660200080589106378,
a[4,3]=-.4.00647866821362550479220191557542444839974791905233931957992861664352492664008930950,
a[5,1]=.847766844780665006269151213101289163348035079739224717380749793482681804846048745312e-1,
a[5,2]=-.2.443928940807501691755389933887625601412449045086435864762969664062854081147152228062,
a[5,3]=2.631456277778367516790671161106440151548475330100155852075179281189837764281765692200,
a[5,4]=.5247059453284763122115578042949767993099011040223243183583532772789009603010474420578,
a[6,1]=.7225995368189859305299905956541324249767281480413725995286879844289944261765025665040e-1,
a[6,2]=9.516272353088938635942728217598994692947741104408877408685737368235625939264822709610,
a[6,3]=-.8.4676523955049948883125735239334776790698654715856595997369562606773887050649070127712,
a[6,4]=-.9878911374935855710712999585545825468517218439959501157763744682322440011952674424511,
a[6,5]=.8670112262277432252013079207249514021049626406395314445073309083276056699618646039028,
a[7,1]=.9193781026315066509020507382422719076650626453057427163695926180790822706243187432582e-1,
a[7,2]=1.156535310004042652442630445260361167016899375767152328888065904869243192778417133612,
a[7,3]=-.78133572807819174425748316517460897558826443327725931110806674742741753121945487541597,
a[7,4]=.197624430807012922551372003130141427696393297855439005163999635744734184538785998772,
a[7,5]=.2716401129021940378395164785269547339747753598120586744252352539821683212723212468596,
a[7,6]=.6359806410179146632799396725005174106044410338193013634200709004038215262748989948510e-1,

b[1]=.9193781026315066509020507382422719076650626453057427163695926180790822706243187432582e-1,
b[2]=1.156535310004042652442630445260361167016899375767152328888065904869243192778417133612,
b[3]=- .78133572807819174425748316517460897558826443323772593118086674742741753121945487541597,
b[4]=.19762443080701292255571372003130141427696393297855439005163999635744734184538785998772,
b[5]=2.1764601129021940378395164785269547339747753598120586744252352539821683212723312468596,
b[6]=.6359806410179146632799396725005174106044410338193013634200709004038215262748989948510e-1,

$b^*[1]=.9216759833961909194544327463126030278322001356391757924238297460444656359020335598376e-1,$
 $b^*[2]=1.131756566522320608485771618471414231000811425558430640037287721214773044875915623949,$
 $b^*[3]=-.7597549114127956052239072871265591842938768107529748269893478936197366723952727266674,$
 $b^*[4]=.2055730782945347272063965557030103623259004844333756140913473187600520022292624970940,$
 $b^*[5]=.2647674310055701921657842784065774056669208954222630333096020574966383669729215651734,$
 $b^*[6]=.4049023725075098542051155991429688251702399177498796030872782154382669472696968446748e-1,$
 $b^*[7]=.25e-1.$