A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

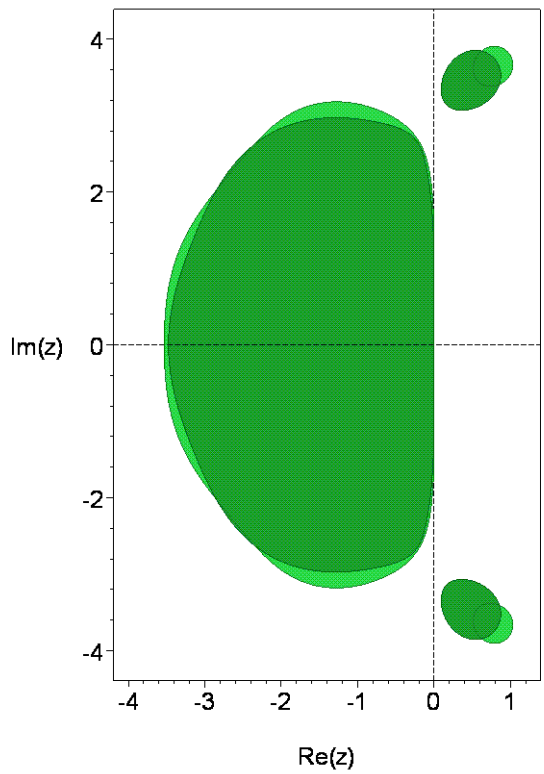
This scheme is constructed using an algorithm of S.N. Papakostas and G. PapaGeorgiou.
See: A Family of Fifth-order Runge-Kutta Pairs, by S.N. Papakostas and G. PapaGeorgiou,
Mathematics of Computation,Volume 65, Number 215, July 1996, Pages 1165-1181.

The nodes of the scheme are:

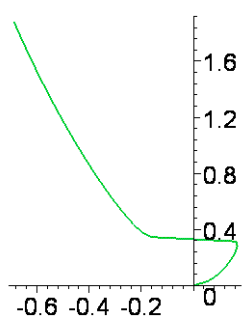
$$c_2 = \frac{6}{41},\ c_3 = \frac{28}{85},\ c_4 = \frac{23}{24},\ c_5 = \frac{127}{128},\ c_6 = 1,\ c_7 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is: $0.4577823251 \times 10^{(-4)}$.
The principal error norm of the order 4 embedded scheme is: $0.3932813110 \times 10^{(-2)}$.
The maximum magnitude of the linking coefficients is: 27.00995851.
The 2-norm of the linking coefficients is: 41.70087166.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-3.5310, 0]$ and $[-3.47595, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0, 0.3358].

The coefficients are as follows:
c[2]=6/41,
c[3]=28/85,
c[4]=23/24,
c[5]=127/128,
c[6]=1,
c[7]=1,

a[2,1]=6/41,
a[3,1]=-896/21675,
a[3,2]=8036/21675,
a[4,1]=145641773/32514048,
a[4,2]=-20539483/2322432,
a[4,3]=57690095/10838016,
a[5,1]=14581996531021/2609293295616,
a[5,2]=-447250011137/40516976640,
a[5,3]=313687066329083/48517728960512,
a[5,4]=-321444747/13899919360,
a[6,1]=189914837873/32509748544,
a[6,2]=-9172151/794976,
a[6,3]=230744117664595/34322819038144,
a[6,4]=-7592400/453818911,
a[6,5]=-143196160/14084042063,
a[7,1]=48635/490728,
a[7,2]=0,
a[7,3]=14731016375/29531467896,
a[7,4]=2553984/383617,
a[7,5]=-964689920/35716083,
a[7,6]=1183/57,

b[1]=48635/490728,
b[2]=0,
b[3]=14731016375/29531467896,
b[4]=2553984/383617,
b[5]=-964689920/35716083,
b[6]=1183/57,

b*[1]=22292993/247186704,
b*[2]=0,
b*[3]=7871636235625/14875422257328,
b*[4]=2279881296/676316771,
b*[5]=-689130766336/62967454329,
b*[6]=1323777/167485,
b*[7]=1/20.

Version: 15 Dec 2012, Peter Stone