A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

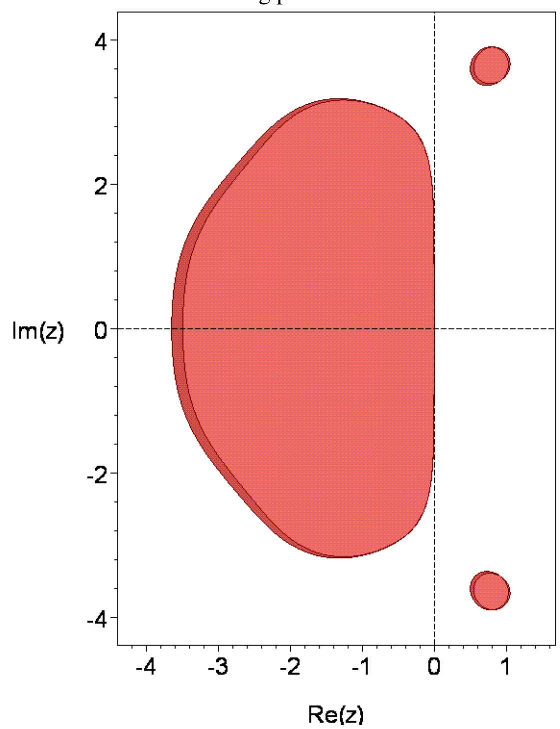
The scheme considered here was constructed to have a small principal error norm.
The nodes of the scheme are:

$$c_2 = \frac{113}{520},\ c_3 = \frac{339}{1040},\ c_4 = \frac{362}{385},\ c_5 = \frac{74675}{76446},\ c_6 = 1,\ c_7 = 1.$$

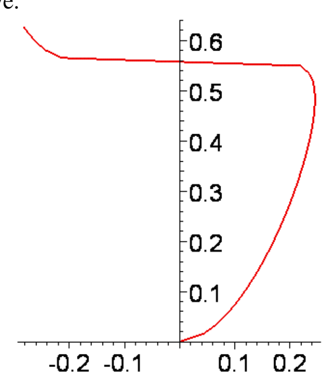
The principal error norm, that is, the 2-norm of the principal error terms is: $0.9524155544 \times 10^{(-4)}$.
Note: The scheme satisfies 9 of the principal error conditions.

The principal error norm of the order 4 embedded scheme is: $0.4178760288 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 8.243437954.
The 2-norm of the linking coefficients is: 19.64831617.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-3.4885, 0]$ and $[-3.6434, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: [0, 0.5593].

The coefficients are as follows:
c[2]=113/520,
c[3]=339/1040,
c[4]=362/385,
c[5]=74675/76446,
c[6]=1,
c[7]=1,

a[2,1]=113/520,
a[3,1]=339/4160,
a[3,2]=1017/4160,
a[4,1]=383465172634/262326144465,
a[4,2]=-492557179504/87442048155,
a[4,3]=1340861078336/262326144465,
a[5,1]=7446772917673199818532687775/3962224643677852581872635668,
a[5,2]=-3309653300816469954230000/456057164327561300860011,
a[5,3]=42956959550319619990258424000/67304460254297169219634173669,
a[5,4]=-47473793128884047395373755/16960670103501066128911214,
a[6,1]=5392761559712859922223/2522214422172948870000,
a[6,2]=-232024529584640/28146573174567,
a[6,3]=7676759785458330278221369600/10747405710798214821737607900,
a[6,4]=4687712616642401500/6766526987321709332,
a[6,5]=-29560223659117229526723660/6644025686131129840583852500,
a[7,1]=899476577/9163966650,
a[7,2]=0,
a[7,3]=149160856179712000/302464617376127781,
a[7,4]=2881033387106875/661323480744471,
a[7,5]=-30337141152778648989786/36832425095777807053250,
a[7,6]=734763181/171322998,

b[1]=899476577/9163966650,
b[2]=0,
b[3]=149160856179712000/302464617376127781,
b[4]=2881033387106875/661323480744471,
b[5]=-30337141152778648989786/36832425095777807053250,
b[6]=734763181/171322998,

b*[1]=785543506000573910/808396645206143750,
b*[2]=0,
b*[3]=158968210646008115823411200/320181936035860529963746500,
b*[4]=2886939339369119809875100/700061495614299505531500,
b*[5]=-10710470716713883975530918103020/1392497844442879848860882093750,
b*[6]=102805646914654510/2590835708462100,
b*[7]=3/500.

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